CONTRACT PRICING REFERENCE GUIDE

VOLUME II

QUANTITATIVE TECHNIQUES FOR CONTRACT PRICING

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PREFACE

CONTRACT PRICING REFERENCE GUIDES

The Air Force Institute of Technology and the Federal Acquisition Institute jointly prepared a series of contract pricing reference guides for pricing and contract personnel. These guides, listed below, are referenced in FAR 15.805-1.

Volume I - Price Analysis

Volume II - Quantitative Techniques for Contract Pricing & Glossary of Terms

Volume III - Cost Analysis

Volume IV - Advanced Issues in Contract Pricing

Volume V - Federal Contract Negotiation Techniques

The five volumes are part of a curriculum of courses used to help contracting personnel become proficient in the performance of the duties and tasks associated with their particular jobs.

USING THE CONTRACT PRICING REFERENCE GUIDE IN THE CLASSROOM

Classroom Learning Objectives (CLOs)	The classroom learning objectives are listed at the beginning of each chapter. The text/reference provides you with the information necessary to accomplish those objectives. Likewise, the classroom instruction and exercises are designed to help you attain those objectives. Most of the objectives are written in terms of your performance of a duty or task. For example, the Text/Reference provides a step by step guide to performing the duties. In the classroom, you will have opportunities to practice performance of the duties. You will use the Text/Reference as your guide, using such instructional techniques as interactive viewgraphs and case studies.
Interactive Viewgraphs	An interactive viewgraph is a slide on the overhead projector that requires a response from the class. For example, if the instructor is showing a decision table, the “then” side would be empty and you would help fill in the answers. Or perhaps the slide asks a particular question about a list of conditions shown on the slide.
Case Studies	Case studies are written as scenarios or stories about particular procurement situations. There are several questions that follow the scenarios relating to the case and the particular lesson. Sometimes you will have to use information in the Text/Reference to complete a case study.
Reading Assignments	You are responsible for <u>all</u> assigned readings from the chapters. You will spend minimal time listening to lectures. Our philosophy is that you learn best by doing the tasks under simulated conditions.
Testing	There will be testing. Test items are taken <u>only</u> from the assigned readings. All test questions were developed to verify the learning acquired from the course learning objectives which appear on the first page of each chapter in the Text/Reference.

USING THE CONTRACT PRICING REFERENCE GUIDE AT YOUR JOB SITE.

The Text/Reference was developed for use at your job site as well as in the classroom. Its step by step approach, FAR references, structured writing and index are all designed for the easy and quick retrieval of information about the contracting process. Each Text/Reference is “dated” by indicating which Federal Acquisition Circular (FAC) of the FAR system it is current through. This lets you know exactly how up to date it is. You may contract the FAI for updates or annotate your own copy as FAR policies change.

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This 1996 version of Quantitative Techniques for Contract Pricing which represents Volume II of the five (5) volume Contract Specialist Reference Guides, were prepared by:

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We also acknowledge all previous course directors, instructors, and field experts who have made tremendous contributions to this course since its inception over 30 years ago. This reference guide is a joint effort, pursuant to FAR 15.805-1, between the Federal Acquisition Institute (FAI) and the Air Force Institute of Technology (AFIT).

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QUANTITATIVE TECHNIQUES FOR CONTRACT PRICING

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CHAPTER 2 Cost-Volume-Profit Analysis

CHAPTER 3 Statistical Analysis

CHAPTER 4 Cost Estimating Relationships

INTRODUCTION

Quantitative Techniques for Contract Pricing

Learning Objective

At the end of this Chapter

At the end of this chapter, you will be able to:

Classroom Learning Objective I/1
Correctly identify potential uses of quantitative techniques in contract pricing.

I.0 Chapter Introduction

Chapter Introduction

To perform either price or cost analysis, you must be able to compare and analyze numbers. To make even the most basic comparisons, you must have an understanding of what quantitative analysis techniques should be applied in making those comparisons and how those comparisons should be made.

Remember the old saying, “Figures don't lie but liars figure.” A technique applied improperly may be used to negotiate or justify an unreasonable price. To assure that prices are fair and reasonable, you must be able to determine:

- When a technique should be used and when it should not be used.
- How an analysis technique should be used and how it should not be used.

Use in Price Analysis

Why use quantitative techniques in price analysis?

In price analysis, you are attempting to determine whether an offered price is fair and reasonable. To do that, you must consider all available information about the product and price. Typically, much of the available information requires quantitative adjustments before it can be used in price analysis. The information may be a price:

- from an earlier purchase,
- for a different quantity,
- from a different production period,
- for a similar, but not identical, item,
- from a different geographic area, or
- for delivery under different business terms than proposed for the current contract action.

To make these prices comparable to the low offer, you may need to use one or more of the quantitative techniques covered in this text.

I.0 Chpater Introduction (cont)

Use in Cost Analysis

Why use quantitative techniques in cost analysis?

In cost analysis, you are developing negotiation positions concerning the allowability of the costs proposed by the offeror. To develop negotiation positions, you must understand the rules for proposal preparation and cost allowability. You must also be able to use the basic quantitative techniques used in cost estimation and analysis. To do this, you must be able to:

- Determine if the offeror used appropriate techniques in estimate development.
- Determine if the offeror properly applied the techniques used in estimate development.
- Understand how other government analysts used a particular technique to confirm or refute offeror estimates.
- Develop your own independent estimate using one or more quantitative techniques.

Additional Information on Quantitative Techniques

The chapters in this text introduce the major quantitative techniques used in cost/price estimating and analysis. However, this text is just an introduction. If you wish to learn more about a particular quantitative technique or its use in contract pricing, consult one or more of the following:

- Defense Contract Audit Agency Audit Manual (DCAAM).
- E-Z-Quant Quantitative Methods for Auditors (DCAAP 7641.91)
- Commercial forecasting texts

CHAPTER 1

Using Price Index Numbers

Learning Objectives

At the End of This Chapter

At the end of this chapter, you will be able to:

Classroom Learning Objective 1/1
Correctly use price index numbers in estimating and analyzing contract cost or price.

1.0 Chapter Introduction

In This Chapter

In this chapter, you will learn to use price index numbers to make the price adjustments necessary to analyze price and cost information collected over time.

Price Index Number Definition

Probably the most commonly used quantitative measure used in contract pricing is the price index number. Price index numbers measure changes in prices over time.

Any discussion of index numbers in contract pricing normally refers to price indexes. However, other index numbers could be used in contract pricing, particularly measures of productivity.

1.0 Chapter Introduction (cont)

Simple and Aggregate Price Index Numbers

Price index numbers can indicate price changes for one or several related items or services over a period of time.

- **Simple index numbers calculate price changes for a single item over time.**

  Index numbers are more accurate if they are constructed using actual prices paid for a single commodity, product or service rather than the more general aggregated index.

- **Aggregate index numbers calculate price changes for a group of related items over time.**

  Aggregated indexes permit analysis of price changes for the group of related products, such as price changes for apples, oranges, plywood, or nails. An example of an aggregate price index is the *Producer Price Index* (Bureau of Labor Statistics) that provides information the changes in the wholesale price of products sold in the United States over a given period of time.

1.1 Identifying Situations for Use

Situations for Use

You can use price index numbers to:

- **Inflate/deflate prices or costs for direct comparison**

  You can use price index numbers to estimate/analyze product cost or price today using the cost or price of the same or a similar product in the past.

- **Inflate/deflate prices or costs to facilitate trend analysis**

  You can use index numbers to facilitate trend or time series analysis of costs or prices by eliminating or reducing the effects of inflation so that the analysis can be made in constant-year dollars (dollars free of charges related to inflation/deflation).

- **Estimate project price or cost over the period of contract performance**

  Prices and costs of future performance are not certain. One effect that you must consider is the changing value of the dollar. You can use index numbers to estimate and negotiate future costs and prices.

- **Adjust contract price or cost for inflation/deflation**

  When price changes are particularly volatile, you may need to include an economic price adjustment clause in the contract. The use of index numbers is one of the most popular methods used to identify and define price changes for economic price adjustment.

1.2 Constructing Price Index Numbers

Steps in Price Index Number Development

If your activity repeatedly buys the same types of services or supplies, consider developing your own price indices to track trends in price over time. This section will demonstrate the procedures for developing a simple price index. To develop an aggregate index, follow the same basic steps using data from the various products selected for index development.

There are four steps to developing a simple price index number:

Step 1. Collect data for each period.

Step 2. Select an appropriate base period.

Step 3. Divide each period price by the base-period price.

Step 4. Multiply by 100 to produce the index number.

Step 1. Collect Data for Each Period

For each index period, collect average price data for the product, commodity, or service. For example, assume the following average yearly prices for a hoist:

Year	**19X4**	**19X5**	**19X6**	**19X7**	**19X8**
Price	$84.12	$90.84	$95.06	$101.97	$107.32

Step 2: Select an Appropriate Base Period

Select a base period appropriate for the data available. In this case, we will use the 19X4 price, $84.12.

SELECT BASE PERIOD		
A	**B**	**C**
Year	**Average Annual Price**	**19X4 Base Price**
19X4	$84.12	$84.12
19X5	$90.84	$84.12
19X6	$95.06	$84.12
19X7	$101.97	$84.12
19X8	$107.32	$84.12

1.2 Constructing Price Index Numbers (cont)

Steps in Price Index Number Development (cont)

Step 3. Divide each period price by the base-period price.

Divide each period price (Column B) by the base-period price (Column C). The result is a price relative. A price relative is the relationship of the price in any period to the base period price. For example, the table below shows that the price in 19X6 is equal to 1.13 times the price in 19X4.

CALCULATE A PRICE RELATIVE				
A	**B**	**C**	**D**	**E**
Year	**Average Annual Price**	**19X4 Base Price**	**Price Relative Calculation**	**Price Relative**
19X4	$84.12	$84.12	$84.12 / $84.12	1.000
19X5	$90.84	$84.12	$90.84 / $84.12	1.080
19X6	$95.06	$84.12	$95.06 / $84.12	1.130
19X7	$101.97	$84.12	$101.97 / $84.12	1.212
19X8	$107.32	$84.12	$107.32 / $84.12	1.276

1.2 Constructing Price Index Numbers (cont)

Steps in Price Index Number Development (cont)

Step 4: Convert to an Index Number

Convert to an index number (Column F) by multiplying each price relative (Column E) by 100. Normally, we round index numbers to the nearest tenth.

CALCULATE PRICE INDEX					
A	**B**	**C**	**D**	**E**	**F**
Year	**Average Annual Price**	**1987 Base Price**	**Price Relative Calculation**	**Price Relative**	**Index Number**
19X4	$84.12	$84.12	$\frac{\$84.12}{\$84.12}$	1.000	100.0
19X5	$90.84	$84.12	$\frac{\$90.84}{\$84.12}$	1.080	108.0
19X6	$95.06	$84.12	$\frac{\$95.06}{\$84.12}$	1.130	113.0
19X7	$101.97	$84.12	$\frac{\$101.97}{\$84.12}$	1.212	121.2
19X8	$107.32	$84.12	$\frac{\$107.32}{\$84.12}$	1.276	127.6

1.3 Selecting a Price Index for Analysis

Sources of Published Indexes

You may not have the time or data required to construct the price indexes that you need for price or cost analysis. Fortunately, there are many sources of previously constructed price indexes, that you can use to estimate price changes. These sources include:

- Bureau of Labor Statistics
- Other Government agencies
- Government contracting organizations
- Commercial forecasting firms
- Industry
- Newspapers

Points to Consider in Index Selection

Use published indexes carefully, since a published index will usually not exactly fit the pattern of price changes for the product or service that you are analyzing. The data are usually not from a specific contractor or location, but represent national or regional averages. Nevertheless, preconstructed index numbers offer a practical alternative to the costly and time-consuming task of developing index numbers from basic cost data.

When you use published indexes, choose the index series that best fits your specific analysis effort. Usually, the closer the chosen index series relates to the item that you are pricing, the more useful the number will be in your analysis.

If you are buying a finished good, indices representing raw materials and purchased components will not necessarily provide an accurate basis for projecting prices for the finished good may also be strongly influenced by trends in direct labor, cost of capital, etc. Accuracy can be improved through use of a weighted average index which represents changes in both labor and material elements of price. Many contracting organizations develop weighted average indexes for major products or major groups of products.

1.3 Selecting a Price Index for Analysis (cont)

Indexes from the Bureau of Labor Statistics

The Government collects and publishes vast amounts of data on prices. Four of the best known sources of index numbers are published by the Bureau of Labor Statistics (BLS):

- **Producer Price Indexes**

  Probably the best known and most frequently used source of price index numbers for material pricing is the *Producer Price Indexes (PPI)* published monthly by the U.S. Department of Labor, Bureau of Labor Statistics (BLS). The indexes report monthly price changes at the producer/wholesale level for 15 major commodity groups:

PRODUCER PRICE INDEXES COMMODITY GROUPS	
Commodity Code	**Commodity Description**
01	Farm Products
02	Processed Foods and Feeds
03	Textile Products and Apparel
04	Hides, Skins, Leather, and Related Products
05	Fuels and Related Products and Power
06	Chemicals and Allied Products
07	Rubber and Plastic Products
08	Lumber and Wood Products
09	Pulp, Paper, and Allied Products
10	Metals and Metal Products
11	Machinery and Equipment
12	Furniture and Household Durables
13	Nonmetallic Mineral Products
14	Transportation Equipment
15	Miscellaneous Products

1.3 Selecting a Price Index for Analysis (cont)

Indexes from the Bureau of Labor Statistics (cont)

- **Consumer Price Index Detailed Report**

  The consumer price index (CPI), published monthly in the *Consumer Price Index Detailed Report*, reports on changes in consumer prices for a fixed mix of goods selected from the following categories:
  - ◊ food
  - ◊ clothing
  - ◊ shelter and fuels
  - ◊ transportation
  - ◊ medical services

  You **should normally not use the CPI in adjusting material prices** because the CPI reflects retail rather than wholesale price changes. However, the CPI can be of value in pricing services when labor rate increases are linked to changes in the CPI.

- **Monthly Labor Review**

  The Monthly Labor Review includes selected data from a number of Government indexes, including:
  - ◊ Employment Cost Index
  - ◊ Consumer Price Index
  - ◊ Producer Price Indexes
  - ◊ Export Price Indexes
  - ◊ Import Price Indexes

  That data and other information presented in the publication can prove useful in analyzing the price of contracts, such as service contracts, where direct labor is a significant part of contract price.

- **Labor Hour and Earnings Report**

  The Labor Hour and Earnings Report presents information on the hours worked and an earnings index for various classes of labor. Like the *Monthly Labor Review*, the report can be very useful in pricing contracts in which direct labor is a significant part of the contract price.

1.3 Selecting a Price Index for Analysis (cont)

Indexes from Other Government Agencies

Data on contract prices are also available from agencies other than the Bureau of Labor Statistics. The most notable are the Federal Reserve System and the Bureau of Economic Analysis.

- **Federal Reserve System**.

  The Board of Governors publishes the *Federal Reserve Bulletin*, which includes economic indexes and data on business, commodity prices, construction, labor, manufactures, and wholesale trade. Each bank in the system publishes information each month with special reference to its own Federal Reserve District.

- **Bureau of Economic Analysis Publications**.

  The Bureau of Economic Analysis, Department of Commerce, publishes the *Survey of Current Business* and the *Business Conditions Digest*. The *Survey of Current Business* provides general information on trends in industry and the business outlook. It furnishes economic indexes on business, construction, manufactures, and wholesale trade. The *Business Conditions Digest* presents almost 500 economic indicators in a form convenient for analysis, as well as different approaches to the study of current business conditions and business prospects, including leading economic indicators.

Indexes from Government Contracting Organizations

Many Government contracting organizations have teams of analysts who develop indexes that are particularly applicable to the organizations’ specific contracting situations. These indexes may be developed from raw price data, or they may be developed as weighted averages of published indexes.

1.3 Selecting a Price Index for Analysis (cont)

Indexes from Commercial Forecasting Firms	Numerous commercial indexes are available for use in contract price analysis. While most Government indexes only report historical price changes, many commercial indexes also forecast future price movement. In situations where forecasts are necessary, commercial indexes may prove particularly useful. Before using such indexes, examine their development and consult with auditors, technical personnel, and other contracting professionals to assure that they are applicable in your analysis situation.
Indexes from Industry	Industry and trade publications frequently provide general forecasts of economic conditions and price changes anticipated in the industry. To identify which publications have economic information relevant to a particular product, ask Government technical personnel. Offerors can also assist you in the identification of appropriate publications. However, be sure to verify with Government personnel the appropriateness of sources of information recommended by offerors.
Indexes from Newspapers	Publications, such as local, national, and financial newspapers, provide valuable forecasts of price changes in specific industries. The information reported is normally data provided by the Government, economic forecasting firms, or industry groups.

1.4 Adjusting Price/Cost for Analysis

Section Introduction

In this Section

In this section, you will learn how to use price index numbers to adjust prices and costs for analysis.

Introduction

The changing value of the dollar over time can complicate comparisons and other analysis using price or cost information collected over time. You can use price indexes to adjust prices/costs to compensate for inflation or deflation and facilitate comparisons and other analysis.

1.4 Section Introduction (cont)

Calculate Relative Price Change Between Two Periods

Index numbers indicate the percentage change in price relative to the base year. For example, the table below shows that the average product price increased by 23.2 percent between 19X4 and 19X9.

YEAR	PRODUCT INDEX
19X4	100.0
19X5	105.3
19X6	112.0
19X7	116.5
19X8	119.3
19X9	123.2

To adjust prices for inflation or deflation, you must be able to do more than determine how prices have changed relative to the base year. You must be able to determine how prices changed between any two time periods. For example, looking at the table above, how did prices change between 19X6 and 19X9? To calculate the percentage price change between any two time periods, you must follow the same procedure that you would follow if you had actual price data; you must divide.

$$\frac{\text{Index in 19X9}}{\text{Index in 19X6}} = \frac{123.2}{112.0} = 1.10$$

Based on the price index and this calculation, you could estimate that product prices in 19X9 were 1.10 times the prices in 19X6 or 10.0 percent more than the prices in 19X6.

1.4 Section Introduction (cont)

Estimating Price/Cost Using Index Numbers

You can use index numbers to adjust prices or costs from any time period for inflation or deflation. For example, the calculation above demonstrated that product prices increased 10.0 percent between 19X6 and 19X9. If you knew that the price for an equipment item in 19X6 was $1,000, you could estimate that the price should be 10.0 percent higher in 19X9. That would result in a price estimate of $1,100 for 19X9.

These calculations can be formalized into a simple equation using either the Ratio Method or the Price Adjustment Formula Method described below.

Ratio Method. The Ratio Method uses an equation in the form of a simple ratio to make the price adjustment.

$$\frac{\text{Index in } T_2}{\text{Index in } T_1} = \frac{\text{Price Estimate for } T_2}{\text{Price in } T_1}$$

Where:

T_1 = Time Period 1 (The time period from which you obtained the price/cost you wish to use as a base for your estimate.)

T_2 = Time Period 2 (The time period for which you intend to estimate price/cost.

Example:

$$\frac{\text{Index in 19X9}}{\text{Index in 19X6}} = \frac{\text{Price Estimate in 19X9}}{\text{Price in 19X6}}$$

$$\frac{123.2}{112.0} = \frac{\text{Price Estimate for 19X9}}{\$1,000}$$

$$123.2 \text{ x } \$1,000 = 112.0 \text{ x Price Estimate for 19X9}$$

$$\$123,200 = 112.0 \text{ x Price Estimate for 19X9}$$

$$\$1,100 = \text{Price Estimate for 19X9}$$

1.4 Section Introduction (cont)

Estimating Price/Cost Using Index Numbers (cont)

Price Adjustment Formula Method. The Price Adjustment Formula is a simplification of the Ratio Method described above.

$$\frac{\text{Index in } T_2}{\text{Index in } T_1} \text{ x Price in } T_1 = \text{Price Estimate for } T_2$$

Example:

$$\frac{\text{Index in 19X9}}{\text{Index in 19X6}} \text{ x Price in 19X6} = \text{Price Estimate for 19X9}$$

$$\frac{123.2}{112.0} \text{ x \$1,000} = \text{Price Estimate for 19X9}$$

$$1.10 \text{ x \$1,000} = \text{Price Estimate for 19X9}$$

$$\$1{,}100 = \text{Price Estimate for 19X9}$$

1.4 Section Introduction (cont)

Adjustment Period Selection

When adjusting historical prices for inflation, take care in selecting the period of adjustment. There are two basic methods that you can use in adjusting costs/prices:

- **Adjustment based on period between acquisition dates.**

  This is the method most commonly used to calculate the period of price adjustment, because acquisition dates are readily available.

  For example, an item is being acquired in January 19X2 was last purchased in January 19X1. Using this method, the logical adjustment period would be January 19X1 to January 19X2 -- a year of inflation or deflation.

  If delivery schedules are similar, this method should be satisfactory. However, if delivery schedules are significantly different, you may be over or under the adjustment required.

  For example, if the January 19X1 acquisition provided for delivery in January 19X2 and the January 19X2 acquisition also provided for delivery in January 19X2, allowing for a year of inflation or deflation would likely over estimate the adjustment required. The pricing of the first acquisition should have already considered the anticipated price changes between January 19X1 and January 19X2. Why make a second adjustment for the same price changes?

- **Adjustment based on period between delivery dates.**

  This method for determining the appropriate period of adjustment is probably more accurate for the reasons described above. The problem with applying this method is the collection of accurate information on delivery dates. Application is further complicated by deliveries over an extended period of time.

For smaller dollar material purchases in periods of limited price changes, the differences between acquisition date to acquisition data and delivery date to delivery date adjustment may not be that significant. However, as contract costs/prices increase or cost/price changes become more volatile, selection of the proper adjustment period becomes more important.

Wage rates should always be estimated for the time period in which the work will be performed.

1.4.1 Adjusting Price/Cost for Pricing Comparisons

Introduction

You can use price indexes to develop should-pay estimates of current price or cost based on historical information. These should-pay estimates can be used for a variety of purposes including comparison with an offered price or cost as part of an evaluation of reasonableness.

Steps in Using Price Indexes to Analyze Price/Cost Reasonable-ness

To perform this analysis, follow the steps below:

Step 1. Collect available price/cost data.

Step 2. Select price indexes for adjusting price/cost data.

Step 3. Adjust price/cost for inflation/deflation.

Step 4. Use adjusted price/cost for pricing comparisons.

Example of Using Price Indexes to Analyze Price/Cost Reasonable-ness

Consider the problem of analyzing a contractor's proposed price of $23,000 for a turret lathe to be delivered in 19X8.

Step 1. Collect Available Price/Cost Data

A procurement history file reveals that the same machine tool was purchased in 19X4 at a price of $18,500. Determine whether the 19X8 proposed price is reasonable.

Step 2. Select An Index Series For Adjusting Price/Cost Data

Select or construct an appropriate index series. In this case, you might select a Machinery and Equipment Index as a reasonable indicator of price movement for a turret lathe. You could extract the data from a publication, such as the PPI, or from a similar commercial index.

YEAR	MACHINERY AND EQUIPMENT INDEX
19X2	100.0
19X3	103.3
19X4	106.0
19X6	110.8
19X7	115.0
19X8	121.9

1.4.1 Adjusting Price/Cost for Pricing Comparisons (cont)

Example of Using Price Indexes to Analyze Price/Cost Reasonable-ness (cont)

Step 3. Adjust Price/Cost for Inflation/Deflation

After you have selected an index, you can adjust prices to a common dollar value level. In this case, you would normally adjust the historical 19X4 price to the 19X8 dollar value level. To make the adjustment, you simply use one of the methods already demonstrated.

Using the Ratio Method

$$\frac{\text{Index in 19X8}}{\text{Index in 19X4}} = \frac{\text{Price Estimate in 19X8}}{\text{Price in 19X4}}$$

$$\frac{121.9}{106.0} = \frac{\text{Price Estimate for 19X8}}{\$18,500}$$

$$121.9 \text{ x } \$18,500 = 106.0 \text{ x Price Estimate for 19X8}$$

$$\$2,255,150 = 106.0 \text{ x Price Estimate for 19X8}$$

$$\$21,275 = \text{Price Estimate for 19X8}$$

Using the Formula Adjustment Method.

$$\frac{\text{Index in 19X8}}{\text{Index in 19X4}} \text{ x Price in 19X8} = \text{Price Estimate for 19X8}$$

$$\frac{121.9}{106.0} \text{ x } \$18,500 = \text{Price Estimate for 19X8}$$

$$1.15 \text{ x } \$18,500 = \text{Price Estimate for 19X8}$$

$$\$21,275 = \text{Price Estimate for 19X8}$$

Step 4: Make Direct Price Comparison

Once you have made the adjustment for inflation/deflation, you can compare the offered and historical prices in constant dollars. The offered price/cost is $23,000, but the adjusted historical price/cost is only $21,275. Thus, the offered price/cost is $1,725, or 8.1 percent higher than what you would expect, given the historical data and available price indexes.

1.4.1 Adjusting Price/Cost for Pricing Comparisons (cont)

Example of Using Price Indexes to Analyze Price/Cost Reasonable-ness (cont)

If you look at the percentage price/cost change between the two acquisitions, the difference is even more pronounced. Using the price indexes, you projected an increase from $18,500 to $21,275, or about 15.0 percent. The actual increase was from $18,500 to $23,000, or about 24.3 percent. In this case, you might ask the offeror why the price/cost rose at a rate 62 percent higher than anticipated.

Do not attempt to determine whether a price or cost is reasonable based this type of analysis alone. You must consider the entire contracting situation, including any differences in quantity, quality, delivery requirements, or other contract terms that might significantly affect price. However, the above analysis does raise concern about the reasonableness of the offer.

Also remember that the analysis above is based on 4-year old data. You should generally place less reliance on a comparison based on 4-year old data than you place on a comparison based on more current data.

1.4.2 Adjusting Price/Cost for Further Analysis

Introduction

Often you will make a series of acquisitions over a period of time. Pricing trends may develop but they may be obscured by inflation/deflation. Adjusting prices for inflation/deflation will make it possible to more accurately identify and track these trends.

Steps in Using Price Indexes to Analyze Price/Cost Reasonable-ness

Adjustment for further analysis follows four steps similar to those used for data adjustment that are applied in preparation for direct comparison. The major difference is that several elements of cost/price data must be adjusted to a single time period. After adjustment, data is said to be in constant-year dollars.

Step 1. Collect available price/cost data.

Step 2. Select price indexes for adjusting price/cost data.

Step 3. Adjust prices/costs for inflation/deflation.

Step 4. Apply appropriate analysis technique.

Example of Using Price Indexes to Adjust Prices/Costs for Further Analysis

To illustrate this analysis, consider an offer of $22,500 each for five precision presses in 1991.

Step 1. Collect Available Price/Cost Data.

The organization has purchased five similar presses each year since 1986. The historical unit prices are shown in Column D of the table below. While purchase quantity changes are not present in this situation, unit prices are used to limit the effect of quantity differences on trend analysis. In this case, the only apparent cost/price trend in the unadjusted data is increasing prices.

Step 2. Select Price Indexes For Adjusting Price/Cost Data.

Again, the Machinery and Equipment Index will be used. Annual indexes are presented in Column B of the table below.

1.4.2 Adjusting Price/Cost for Further Analysis (cont)

Example of Using Price Indexes to Adjust Prices/ Costs for Further Analysis (cont)

Step 3. Adjust Prices/Costs For Inflation/Deflation
The adjustment calculation is presented in Column 3 of the table below. Each historical price is adjusted to equivalent prices in 19X7 dollars.

ADJUSTMENT FOR FURTHER ANALYSIS				
A	**B**	**C**	**D**	**E**
Year	**Machinery and Equipment Index**	**Index Adjustment Calculation**	**Historical Prices**	**Adjusted Prices**
19X2	100.0	$\frac{121.9}{100.0}$	$17,391	$21,200*
19X3	103.3	$\frac{121.9}{103.3}$	$17,796	$21,000
19X4	106.0	$\frac{121.9}{106.0}$	$18,087	$20,800
19X5	110.8	$\frac{121.9}{110.8}$	$18,724	$20,600
19X6	115.0	$\frac{121.9}{115.0}$	$19,245	$20,400
19X7	121.9	--	--	?

* Prices in this column are rounded to the nearest dollar.

1.4.2 Adjusting Price/Cost for Further Analysis (cont)

Example of Using Price Indexes to Adjust Prices/Costs for Further Analysis (cont)

After the historical unit prices are adjusted to 19X7 dollars, a trend becomes obvious. In 19X7 dollars, prices have been dropping $200 each year since 19X2. The obvious price estimate is $20,200 for the 19X7 acquisition. That projection is based on the continuation of the historical trend. However, as with direct comparison, analysis based on historical price trends must consider any changes in the contracting situation and their possible affect on contract price. There may also be questions as to what has caused the trend and whether those forces will continue to cause price changes.

Most trends are not so obvious, even after prices have been adjusted to constant-year dollars. Still, techniques such as regression analysis and improvement curve analysis can often be applied to adjusted data to identify clear estimating relationships.

1.5 Identifying Issues and Concerns

Section Introduction

As you perform price/cost analysis, consider the issues and concerns identified in this section, whenever your analysis is based on data collected over time.

Questions to Consider in Analysis

- ***Were prices/costs collected over time adjusted for inflation/deflation?***

  Inflation/deflation can mask underlying price changes. Price indexes should be used to compensate for the effect these general price changes.

- ***Is it reasonable to use the price index series selected?***

  The price index series selected for making the price/cost adjustment should be as closely related to the item being considered as possible. For example, you should not use the Consumer Price Index to adjust for changes in the price of complex industrial electronic equipment.

- ***Are adjustments calculated correctly?***

  Anyone can make a mistake in calculation. Assure that all adjustments are made correctly. This is particularly important when the adjustment is part of a contractor's offer or part of an analysis performed by other Government personnel.

- ***Is the time period for the adjustment reasonable?***

  When adjusting historical prices for inflation, take care in selecting the period of adjustment. There are two basic methods that are used in adjusting costs/prices, period between acquisition dates and the period between delivery dates. The period between acquisition dates is most commonly used because purchase dates are typically more readily available. However, be careful if delivery schedules are substantially different.

- ***Is more than one adjustment made for the same inflation/deflation?***

  For example, it is common for offerors to adjust supplier quotes to consider inflation/between the time when the quote was obtained and the date that the product will be required. This is acceptable unless the supplier already considered the inflation/deflation in making the quote.

- ***How far into the future should you forecast?***
  The farther into the future that you forecast, the greater the risk.

CHAPTER 2

Using Cost-Volume-Profit Analysis

Learning Objective

At the End of this Chapter	At the end of this chapter you will be able to: **Classroom Learning Objective 2/1** Correctly use price cost-volume-profit analysis in estimating and analyzing contract cost or price.

2.0 Chapter Introduction

In This Chapter

This Chapter includes:

Assumptions

When you acquire supplies or services, you expect to pay a smaller price per unit as the purchase quantity increases. You expect contractors to have lower costs per unit as production quantity increases. This general expectation remains the same whether you are buying items specifically built for the Government, or items that are mass-produced for a variety of commercial and Government customers. Cost-volume-profit analysis can be used to analyze the natural relationship between cost, volume, and profit in pricing decisions.

In cost-volume-profit analysis, you consider only short-term operations. The short term may be defined as a period too short to permit facilities expansion or contraction, or other changes that might affect overall pricing relationships

2.0 Chapter Introduction (cont)

Assumptions (cont)

The technique assumes use of the straight line in analysis. While actual price behavior may not follow a straight line, its use can closely approximate actual cost behavior in the short run. If purchase volume moves outside the relevant range of the available data, the straight-line assumption and the accuracy of estimates becomes questionable.

If you know that product variable costs per unit are decreasing as quantity increases, consider using the log-linear improvement curve concept. Improvement curves are particularly useful in limited production situations where you can obtain data on the price of all units sold.

Types of Cost

In the short run, costs can be of three general types:

- **Fixed Cost.** Total fixed costs remain constant as volume varies in the relevant range of production. Fixed cost per unit decreases as the cost is spread over an increasing number of units.

  Examples include: Fire insurance, depreciation, facility rent, and property taxes.

- **Variable Cost.** Variable cost per unit remains constant no matter how many units are made in the relevant range of production. Total variable cost increases as the number of units increases.

  Examples include: Production material and labor. If no units are made, neither cost is necessary or incurred. However, each unit produced requires production material and labor.

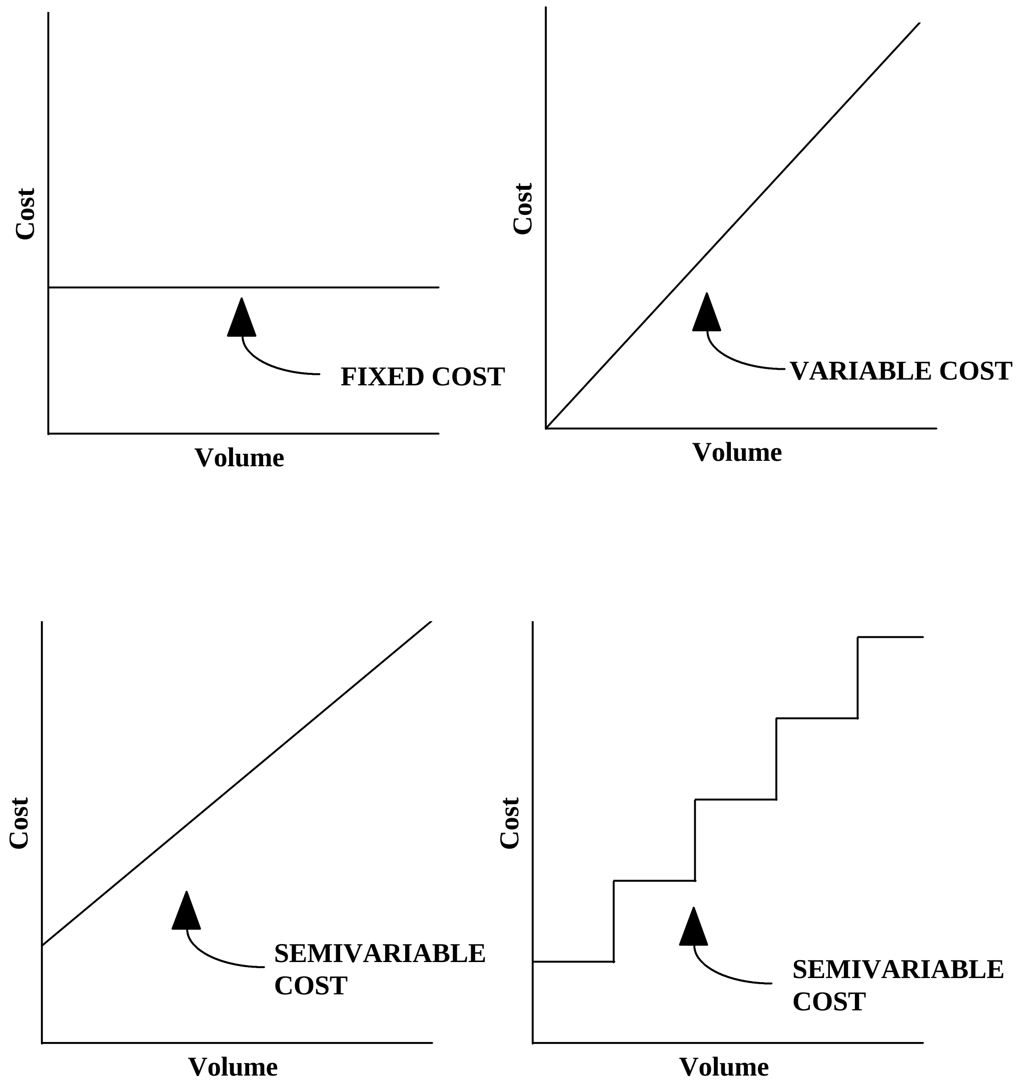
- **Semivariable Cost.** Semivariable costs include both fixed and variable cost elements. Costs may increase in steps or increase relatively smoothly from a fixed base.

  Examples include: Supervision and utilities, such as electricity, gas, and telephone. Supervision costs tend to increase in steps as a supervisor's span of control is reached. Utilities typically have a minimum service fee, with costs increasing relatively smoothly as more of the utility is used.

2.0 Chapter Introduction (cont)

Graphic Depiction of Cost Behavior

The four graphs below illustrates the different types of cost behavior described above:



Profit

Profit is the difference between total cost and revenue. In cost-volume-profit analysis, a loss is expressed as a negative profit. Breaking even, which is neither profit nor loss, is a profit of zero dollars.

2.1 Identifying Situations for Use

Situations for Use

Cost-volume-profit analysis is an estimating concept that can be used in a variety of pricing situations. You can use the cost-volume relationship for:

- **Evaluating item price in price analysis.**

  Cost-volume-profit analysis assumes that total cost is composed of fixed and variable elements. This assumption can be used to explain price changes as well as cost changes. As the volume being acquired increases unit costs decline. As unit costs decline, the vendor can reduce prices and same make the same profit per unit.

- **Evaluating direct costs in pricing new contracts.**

  Quantity differences will often affect direct costs -- particularly direct material cost. Direct material requirements often include a fixed component for development or production operation set-up. As that direct cost is spread over an increasing volume unit costs should decline.

- **Evaluating direct costs in pricing contract changes.**

  How will an increase in contract effort increase contract price? Some costs will increase others will not. The concepts of cost-volume-profit analysis can be an invaluable aid in considering the effect of the change on contract price.

- **Evaluating indirect costs.**

  The principles of cost-volume-profit analysis can be used in indirect cost analysis. Many indirect costs are fixed or semivariable. As overall volume increases, indirect cost rates decline because fixed costs are spread over an increasing production volume.

2.2 Analyzing the Cost-Volume Relationship

In this Section

This section includes:

2.2.1 Algebraic Analysis

Key Assumption	The assumption of linear cost behavior permits use of straight-line graphs and simple linear algebra in cost-volume analysis.

Calculating Total Cost Algebraically

Total cost is a semivariable cost—some costs are fixed, some costs are variable, and others are semivariable. In analysis, the fixed component of a semivariable cost can be treated like any other fixed cost. The variable component can be treated like any other variable cost. As a result, we can say that:

Total Cost = Fixed Cost + Variable Cost

or using symbols:

$C = F + V$ where: C = Total Cost
F = Fixed Cost
V = Variable Cost

Total variable cost depends on two elements:

1. Variable Cost per Unit
2. Quantity Produced

$V = V_U(Q)$ where: V_U = Variable Cost per unit
Q = Quantity Produced

Substituting this information into the basic Total Cost Equation, we have the equation used in cost-volume analysis:

$$C = F + V_U (Q)$$

Example of Calculating Total Cost Algebraically

If you know that Fixed Costs are $500, Variable Cost per Unit is $10, and the Volume produced is 1,000 units, you can calculate the Total Cost of production.

$C = F + V_U(Q)$

$C = \$500 + \$10 (1,000)$

$C = \$500 + \$10,000$

$C = \$10,500$

2.2.1 Algebraic Analysis (cont)

Example of Calculating Variable Cost Algebraically

Given Total Cost and Volume for two different levels of production, and using the straight-line assumption, you can calculate Variable Cost per Unit.

Remember that:

1. Fixed Costs do NOT change no matter what the volume, as long as we are in the relevant range of production. Any change in total cost is the result of a change in total Variable Cost.
2. Variable Cost per Unit does NOT change in the relevant range of production.

As a result, we can calculate Variable Cost per unit (V_U) by:

$$V_U = \frac{\text{Change in Total Cost}}{\text{Change in Volume}}$$

$$V_U = \frac{\text{Total Cost at Point 2 - Total Cost at Point 1}}{\text{Volume at Point 2 - Volume at Point 1}}$$

$$V_U = \frac{C_2 - C_1}{Q_2 - Q_1}$$

You are analyzing an offeror's cost proposal. As part of the proposal the offeror shows that a supplier offered 5,000 units of a key part for $60,000. The same quote offered 4,000 units for $50,000. What is the apparent variable cost per unit?

$$V_U = \frac{C_2 - C_1}{Q_2 - Q_1}$$

$$V_U = \frac{\$60,000 - \$50,000}{5,000 - 4,000}$$

$$V_U = \frac{\$10,000}{1,000}$$

$$V_U = \$10$$

2.2.1 Algebraic Analysis (cont)

Calculating Fixed Cost Algebraically

If you know Total Cost and Variable Cost per Unit for any Quantity, you can calculate Fixed Cost using the basic Total Cost equation.

Example of Calculating Fixed Cost Algebraically

In the previous section, we calculated Variable Cost per Unit given information on two data points. Using the Total Cost for 5,000 units, $60,000; the calculated Variable Cost per Unit, $10; and the Total Cost equation; we can calculate fixed cost.

$$C = F + V_U(Q)$$

where:

C	= $60,000
V_U	= $10
Q	= 5,000

$$\$60,000 = F + (\$10)(5,000)$$

$$\$60,000 = F + \$50,000$$

$$\$60,000 - \$50,000 = F$$

$$\$10,000 = F$$

Developing an Estimating Equation

Now that you know that V_U is $10 and F is $10,000 you can substitute the values into the general Total Cost Equation.

$$C = F + V_U(Q)$$

$$C = \$10,000 + \$10(Q)$$

The result is an equation that can be used to estimate the total cost of any volume in the relevant range between 4,000 and 5,000 units.

Using the Estimating Equation

Estimate the total cost of 4,400 units.

$$C = \$10,000 + \$10(Q)$$

$$C = \$10,000 + \$10(4,400)$$

$$C = \$10,000 + \$44,000$$

$$C = \$54,000$$

2.2.2 Graphic Analysis

Introduction to Graphic Analysis

When you only have two data points, you must generally assume a linear relationship. When you get more data, you can examine the data to determine if there is truly a linear relationship.

You should always graph the data before performing an algebraic analysis.

- Graphic analysis is the best way of developing an overall view of cost-volume relationship.
- Graphic analysis is useful in analyzing cost-volume relationships, particularly, when the cost and volume numbers involved are relatively small.
- Even when actual analysis is performed algebraically you can use graphs to demonstrate cost-volume analysis to others.

2.2.2 Graphic Analysis (cont)

Steps of Graphic Analysis

There are four steps in using graph paper to analyze cost-volume relationships:

Step 1. Determine the scale that you will use.

Volume is considered the independent variable and will be graphed on the horizontal axis. Cost is considered the dependent variable and will be graphed on the vertical axis. The scales on the two axes do not have to be the same. However, on each axis one block must represent the same amount of change as every other block of the same size on that axis. Each scale should be large enough to permit analysis, and small enough to permit the graphing of all available data and anticipated data estimates.

Step 2. Plot the available cost-volume data.

Find the volume given for one of the data points on the horizontal axis. Draw an imaginary vertical line from that point. Find the related cost on the vertical axis and draw an imaginary horizontal line from that point. The point where the two lines intersect represents the cost for the given volume. (If you do not feel comfortable with imaginary lines you may draw dotted lines to locate the intersection.) Repeat this step for each data point.

Step 3. Fit a straight line to the data.

In this section of text, all data points will fall on a straight line. All that you have to do to fit a straight line is connect the data points. Most analysts use regression analysis to fit a straight line when all points do not fall on the line.

Step 4. Estimate the cost for a given volume.

Draw an imaginary vertical line from the given volume to the point where it intersects the straight line that you fit to the data points. Then move horizontally until you intersect the vertical axis. That point is the graphic estimate of the cost for the given volume of the item.

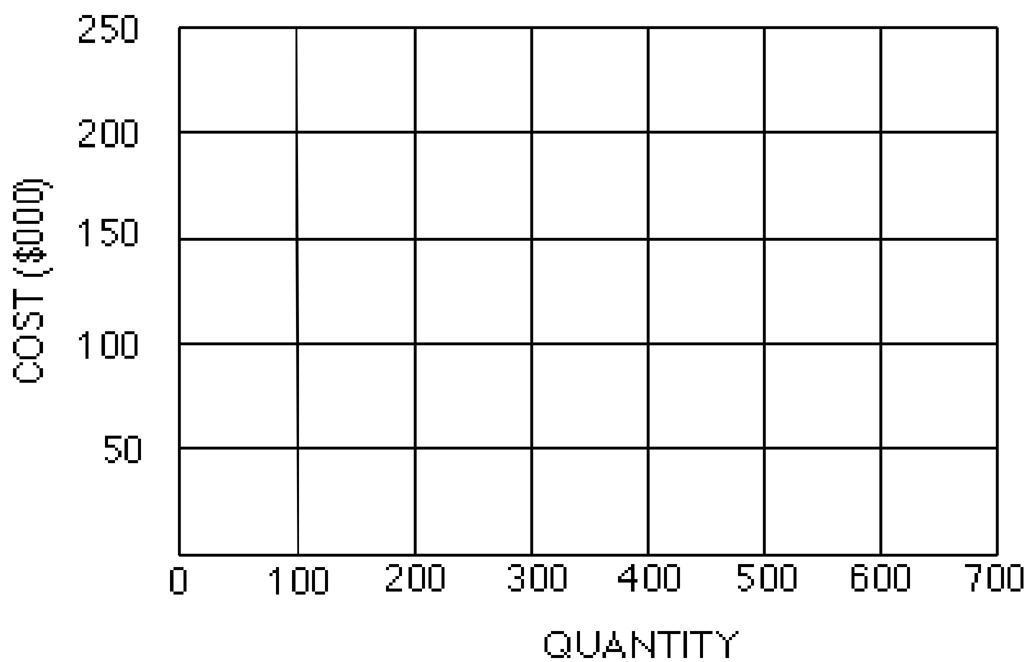
2.2.2 Graphic Analysis (cont)

Example of Graphic Analysis (cont)

The four steps of cost-volume-profit analysis can be used to graph and analyze any cost-volume relationship. Assume that you have been asked to estimate the cost of 400 units given the following data:

UNITS	COST
200	$100,000
500	$175,000
600	$200,000

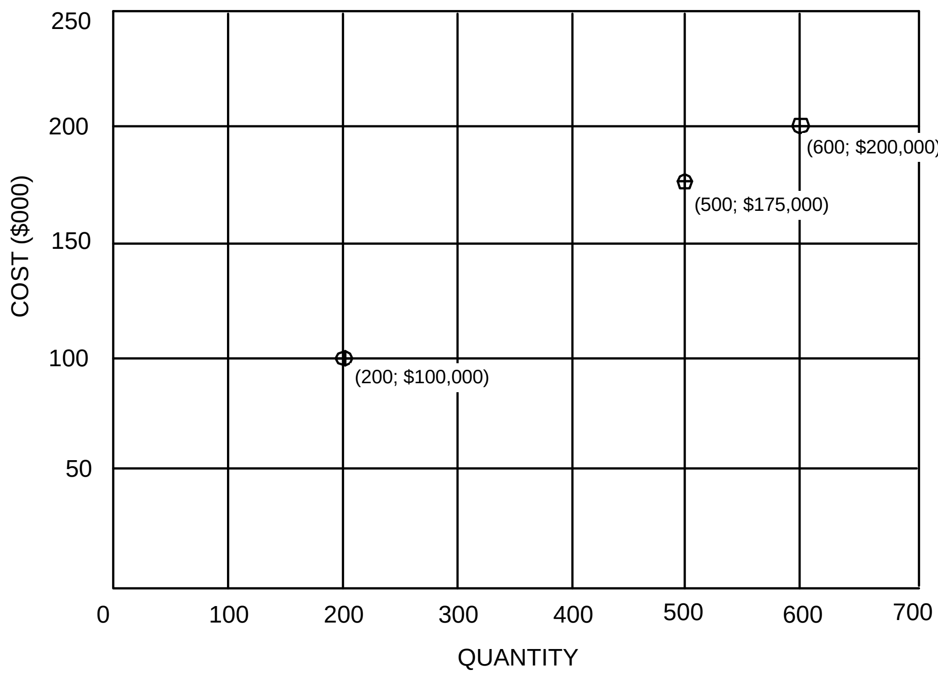
Step 1. Determine the scale that will be used.



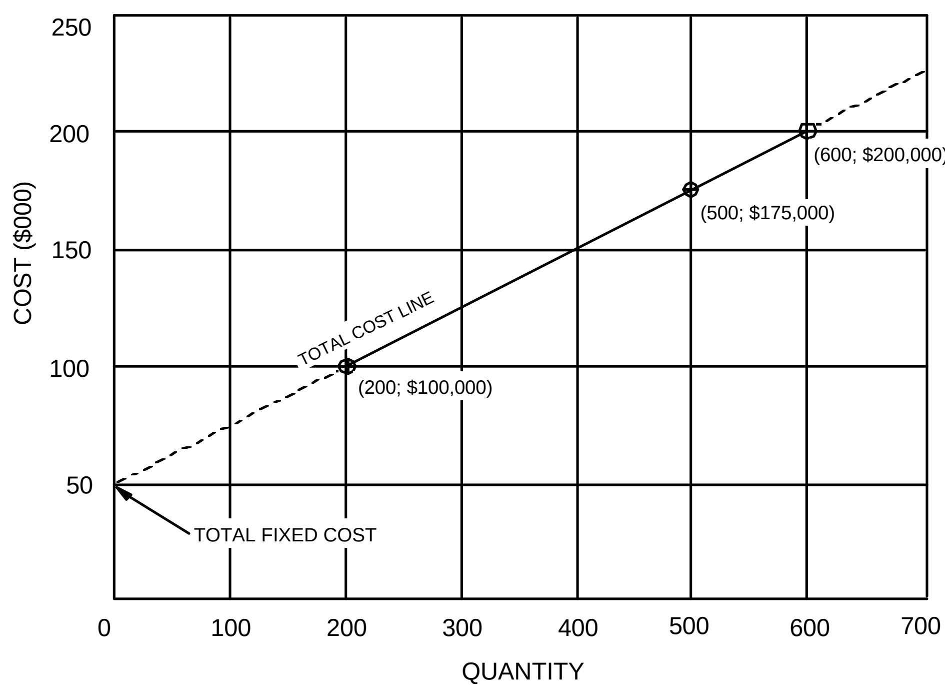
2.2.2 Graphic Analysis (cont)

Example of Graphic Analysis (cont)

Step 2. Plot the available cost-volume data.



Step 3. Fit a straight line to the data.



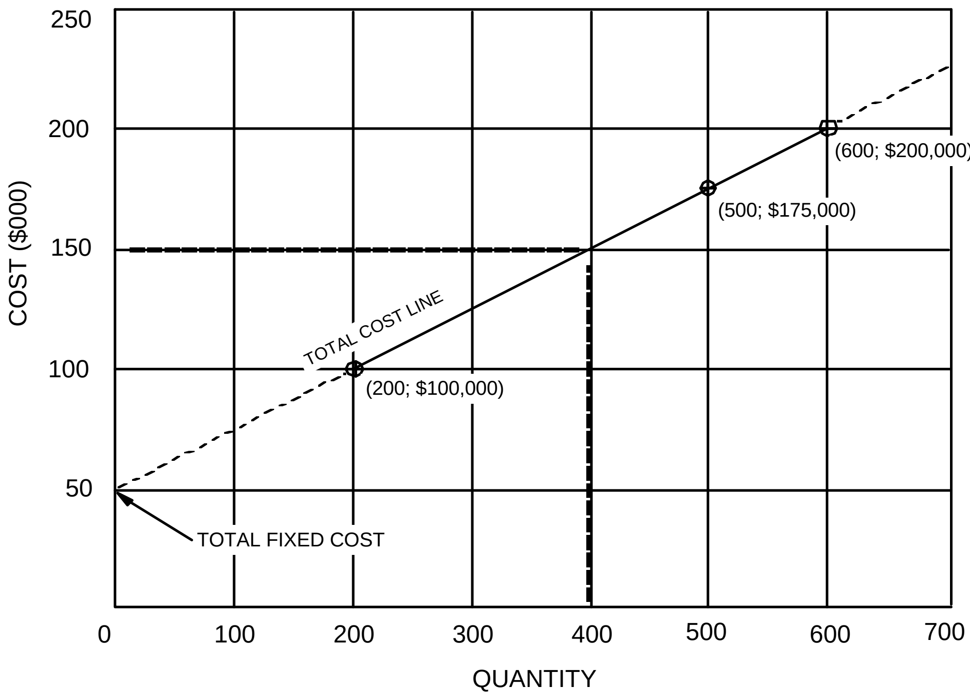
2.2.2 Graphic Analysis (cont)

Graphic Analysis (cont)

Step 4. Estimate the cost for a given volume.

From the graph, you can estimate that the total cost of 400 units will be $150,000.

In addition you can also estimate the cost of making zero units. The cost of making zero units, $50,000, is the fixed cost for this set of data.



Since the graph demonstrates that the data forms a straight line, we can use any two points to calculate the equation of the line. Follow the procedures in the section on Algebraic Analysis of the Linear Cost-Volume Relationship.

2.3 Analyzing the Price-Volume Relationship

Section Introduction

In situations where you do not have offeror cost data, you can use the principles of the cost-volume relationship in price analysis. Price-volume analysis is based on price information that is typically available to the Government negotiator.

Estimating Quantity Price Discounts

Unit prices normally decline as volume increases, primarily because fixed costs are being divided by an increasing number of units. Buyers see these price reductions with increasing volume in the form of quantity discounts.

Quantity discounts complicate the pricing decision, because a price that is reasonable for one volume may not be reasonable for a different volume.

Offers quote quantity discounts because their costs per unit declines as volume increases. As a result, even though offered prices include profit, you can use the cost-volume equation to estimate prices at different quantities.

Example of Estimating a Quantity Price Discount

You know that the unit price for 100 units is $3,000 and the unit price for 500 units is $2,500. You are about to purchase 250 units. How can you use the available information to estimate the price for 250 units?

Approach this question just as though you were being asked to estimate cost given information for two different quantities. The only difference is that you use price data instead of cost data in the analysis.

Step 1. Calculate the variable element.

$$V_U = \frac{C_2 - C_1}{Q_2 - Q_1}$$

$$V_U = \frac{\$1,250,000 - \$300,000}{500 - 100}$$

$$V_U = \frac{\$950,000}{400}$$

$$V_U = \$2,375$$

2.3 Analyzing the Price-Volume Relationship (cont)

Example of Estimating a Quantity Price Discount (cont)

Step 2. Calculate the fixed element using data from the available data points.

$$C = F + V_U(Q)$$
$$\$300,000 = F + \$2,375(100)$$
$$\$62,500 = F$$

Step 3. Develop Estimating Equation.

$$C = F + V_U(Q)$$
$$C = \$62,500 + \$2,375(Q)$$

Step 4. Estimate price for given quantity.

$$C = \$62,500 + \$2,375Q$$
$$C = \$62,500 + \$2,375(250)$$
$$C = \$62,500 + \$593,750$$
$$C = \$656,250 \text{ total price and a } \$2,625 \text{ unit price.}$$

2.4 Analyzing the Cost-Volume-Profit Relationship

Section Introduction

Until now, we have only looked at the cost-volume relationship. Now, we are going to expand that relationship to consider the relationship between cost, volume, and **profit**.

Cost-Volume-Profit Equation

The revenue taken in by a firm is equal to cost plus profit.
That can be written:

Revenue = Total Cost + Profit

We have already seen that total cost (C) is:

$C = F + V_U (Q)$

Using this information, we can rewrite the Revenue equation as:

Revenue = $F + V_U (Q)$ + Profit

In the cost-volume-profit equation, **profit can be positive, negative, or zero** . If profit is negative, we normally refer to it as a loss. If profit is zero, the firm is breaking even, no profit or loss. If we let P stand for profit, we can write the equation:

Revenue = $F + V_U (Q) + P$

Revenue is equal to selling price per unit (R_U) multiplied by volume.

Revenue = $R_U (Q)$

If we assume that the firm makes all the units that it sells, and sells all the units that it makes, we can complete the cost-volume-profit equation:

$R_U (Q) = F + V_U(Q) + P$

2.4 Analyzing the Cost-Volume-Profit Relationship (cont)

Application of the Cost-Volume-Profit Equation	This equation and limited knowledge of a contractor's cost structure can provide you with extremely valuable information on the effect purchase decisions can have on a firm's profitability.
Using the Cost-Volume-Profit Equation to Estimate Selling Price	Given the following product information, a firm prepared an offer for an indefinite quantity contract with the Government for a new product developed by the firm. There are no other customers for the product. In developing the unit price estimate (R_U), the firm used its estimated costs and its best estimate of the quantity that it would sell under the contract.

Fixed Cost	=	$10,000
Variable Cost per Unit	=	$20
Contract Minimum Quantity	=	4,000 units
Contract Maximum Quantity	=	6,000 units
Firm's Best Estimate of Quantity	=	5,000 units
Target Profit	=	$5,000

$$R_U(Q) = F + V_U(Q) + P$$

$$R_U(5{,}000) = \$10{,}000 + \$20(5{,}000) + \$5{,}000$$

$$R_U(5{,}000) = \$10{,}000 + \$100{,}000 + \$5{,}000$$

$$R_U(5{,}000) = \$115{,}000$$

$$R_U(5{,}000) = \frac{\$115{,}000}{5{,}000}$$

$$R_U = \$23.00$$

2.4 Analyzing the Cost-Volume-Profit Relationship (cont)

Using the Cost-Volume-Profit Equation to Estimate Profit

Manager's of the firm wanted to know how profits would be affected if it actually sold the maximum quantity (6000 units) at $23.00 per unit.

$$R_U(Q) = F + V_U(Q) + P$$
$$\$23(6,000) = \$10,000 + \$20(6,000) + P$$
$$\$138,000 = \$10,000 + \$120,000 + P$$
$$\$138,000 = \$130,000 + P$$
$$\$8,000 = P$$

If the firm sells 6,000 at $23.00 per unit, profit will be $8,000. That is a $3,000 increase from the original $5,000 target profit, or an increase of 60 percent. Note that the firm's profit would increase solely because sales were higher than estimated.

Managers were even more concerned about how profits would be affected if they only sold the minimum quantity (4000 units) at $23.00 per unit.

$$R_U(Q) = F + V_U(Q) + P$$
$$\$23(4,000) = \$10,000 + \$20(4,000) + P$$
$$\$92,000 = \$10,000 + \$80,000 + P$$
$$\$92,000 = \$90,000 + P$$
$$\$2,000 = P$$

If the firm sells 4,000 at $23.00 per unit, profit will be $2,000. That is $3,000 less than the original $5,000 target profit, or only 40 percent of the original estimate. Note that the firm's profit would decrease solely because sales were lower than estimated.

2.4 Analyzing the Cost-Volume-Profit Relationship (cont)

Using the Cost-Volume-Profit Equation to Estimate Required Sales

In a final effort to analyze the risk to the firm under the proposed indefinite deliver contract, managers wanted to know the level of sales that would be required for the firm to break even (zero profit).

$$
\begin{aligned}
R_U(Q) &= F + V_U(Q) + P \\
\$23(Q) &= \$10,000 + \$20(Q) + 0 \\
(\$23 - \$20)Q &= \$10,000 \\
\$3(Q) &= \$10,000 \\
Q &= 3333.33 \text{ units}
\end{aligned}
$$

The calculations show that the firm would break even at 3,333.33 units. Assuming that the firm could not sell .33 units, the firm must sell 3,334 units to assure that all costs are covered. Selling 3,333 units would still result in a $.10 loss.

2.4 Analyzing the Cost-Volume-Profit Relationship (cont)

Contribution Income

The difference between revenue and variable cost is contribution income (I). The term contribution income comes from the contribution made to covering fixed costs and profit. If contribution income is positive, increasing sales will increase profits or reduce losses. If contribution income is negative, increasing sales will reduce profits or create greater losses.

Contribution Income = Revenue - Variable Cost

$$I = R_U (Q) - V_U (Q) \quad \text{or}$$

$$I = (R_U - V_U) Q$$

Knowledge of a contractor's cost structure and contribution income can be valuable in analysis of proposed costs.

Contribution Income Example

In evaluating an offeror's proposal for 500 units at $900 each, your analysis reveals the following cost structure:

Fixed Cost = $100,000
Variable Cost per Unit = $1,000

How would this affect your analysis of contract risk?

$I = (R_U - V_U) Q$
I = ($900 - $1,000) (500)
I = (-$100) (500)
I = -$50,000

The contribution income from the sale is a negative $50,000. The firm would be substantially worse off for having made the sale. Unless the firm can offer a positive rationale for such a pricing decision, you must consider pricing as an important factor as you analyze the risk of contract performance.

2.5 Identifying Issues and Concerns

Questions to Consider in Analysis

As you perform price/cost analysis, consider the issues and concerns identified in this section, whenever you use cost-volume-profit analysis concepts.

- **Has the contractor's cost structure changed substantially?**

  Application of cost-volume-profit analysis assumes that the period covered by the analysis is too short to permit facilities expansion or contraction or other changes that might affect overall pricing relationships. If the contractor has substantially changed its cost structure, your ability to use cost-volume-profit analysis may be limited. Examples of possible changes include:

  ◊ Downsizing to reduce fixed costs.

  ◊ Increased investment in automated equipment to reduce variable costs of labor and material.

- **Is the straight-line assumption reasonable?**

  The cost-volume-profit relationship is not usually a straight- line relationship. Instead, it is a curvilinear relationship. A straight-line analysis works as long as the straight line is a good approximation of the cost-volume-profit relationship. Most computer programs designed to fit a straight-line to a set of data provide measures of how well the line fits the data. For example, a regression program will usually provide the coefficient of determination (r^2).

- **Are current volume estimates within the relevant range of available data?**

  If the current business volume is substantially higher or lower than the volumes used to develop the cost-volume-profit equation, the results may be quite unreliable. The contractor should be expected to change the way it does business and its cost structure if volume increases or decreases substantially.

CHAPTER 3

Statistical Analysis

Learning Objectives

At the End of This Chapter

At the end of this chapter you will be able to:

Classroom Learning Objective 3/1
Correctly use descriptive statistics in estimating and analyzing contract cost or price.

3.0 Chapter Introduction

In This Chapter

In this chapter, you will learn to use descriptive statistics to organize, summarize, analyze, and interpret data for contract pricing.

Categories of Statistics

Statistics is a science which **involves collecting, organizing, summarizing, analyzing, and interpreting data in order to facilitate the decision-making process**. These data can be facts, measurements, or observations. For example, the inflation rate for various commodity groups is a statistic which is very important in contract pricing. Statistics can be classified into two broad categories:

- Descriptive Statistics

  Descriptive statistics include a large variety of methods for summarizing or describing a set of numbers. These methods may involve computational of graphical analysis. For example, index numbers are one example of a descriptive statistic. The measures of central tendency and dispersion presented in this chapter are also descriptive statistics, because they describe the nature of the data collected.

- Inferential Statistics

  Inferential or inductive statistics are methods of **using a sample data taken from a statistical population to make actual decisions, predictions, and generalizations related to a problem of interest** . For example, in contract pricing, we can use stratified sampling of a proposed bill of materials to infer the degree it is overpriced or underpriced.

3.0 Chapter Introduction (cont)

Populations and Samples

The terms population and sample are used throughout any discussion of statistics.

- **Population**

  A population is the **set of all possible observations** of a phenomenon with which we are concerned. **For example**, all the line items in a bill of materials would constitute a population. A numerical characteristic of a population is called a **parameter**.

- **Sample**

  A sample is a **sub-set of the population of interest** that is selected in order to make some inference about the whole population. **For example**, a group of line items randomly selected from a bill of materials for analysis would constitute a sample. A numerical characteristic of a sample is called a **statistic**.

In contract pricing, you will most likely use statistics because you do not have complete knowledge of the population or you do not have the resources needed to examine the population data. Because most pricing applications involve the use of sample data, this chapter will concentrate on statistical analysis using sample data. If you are interested in learning about descriptive or inferential analysis of numerical population data, consult a college level statistics text.

Measure of Reliability

Since a sample contains only a portion of observations in the population, **there is always a chance that our inference or generalization about the population will be inaccurate**. Therefore, our **inference should be accompanied by a measure of reliability**. For example, let's assume that we are 90 percent sure that the average item in a bill of materials should cost 85 percent of what the contractor has proposed plus or minus 3 percent. The 3 percent is simply a boundary for our prediction error and it means there is a 90 percent probability that the error of our prediction is not likely to exceed 3 percent.

3.1 Identifying Situations for Use

Situations for Use

Statistical analysis can be invaluable to you in:

- **Developing Government objectives for contract prices based on historical values.**

  Historical costs or prices are often used as a basis for prospective contract pricing. When several historical data points are available, you can use statistical analysis to evaluate the historical data in making estimates for the future. For example, you might estimate the production equipment set-up time based on average historical costs.

- **Developing minimum and maximum price positions for negotiations.**

  As you prepare your negotiation objective, you can also use statistical analysis to develop minimum and maximum positions through analysis of risk. For example, if you develop an objective of future production set-up time based on the average of historical experience, that average is a point estimate calculated from many observations. If all the historical observations are close to the point estimate, you should feel confident that actual set-up time will be close to the estimate. As the differences between the individual historical observations and the point estimate increase, the risk that the future value will be substantially different than the point estimate also increases. You can use statistical analysis to assess the cost risk involved and use that assessment in developing your minimum and maximum negotiation positions.

- **Developing an estimate of risk for consideration in contract type selection.**

  As described above, you can use statistical analysis to analyze contractor cost risk. In addition to using that analysis in developing your minimum and maximum negotiation positions you can use it in contract type selection. For example, if the risk is so large that a firm fixed-price providing reasonable protection to the contractor could also result in a wind-fall profit, you should consider an incentive or cost-reimbursement contract instead.

3.1 Identifying Situations for Use (cont)

Situations for Use (cont)

- **Developing an estimate of risk for consideration in profit/fee analysis.**

  An analysis of cost risk is also an important element in establishing contract profit/fee objectives. The greater the dispersion of historical cost data, the greater the risk accepted by the contractor in prospective contract pricing. As contractor cost risk increases, contract profit/fee should also increase.

- **Streamlining the evaluation of a large quantity of data without sacrificing quality.**

  Statistical sampling is particularly useful in the analysis of a large bill of materials. In particular, the stratified sampling techniques presented in this chapter allow you to examine 100 percent of the items with the greatest potential for cost reduction while using random sampling to assure that there is no general pattern of overpricing smaller value items. The **underlying assumption of random sampling is that a sample is representative of the population from which it is drawn**. Hence, if the sample is fairly priced, the entire stratum is assumed to be fairly priced; if the sample is over-priced, the entire stratum is assumed to be proportionately over-priced.

3.2 Measuring Central Tendency

Measures of Central Tendency

You are about to prepare a solicitation for a product that your office has acquired several times before. Before you begin, you want to know what your office has historically paid for the product. You could rely exclusively on the last price paid, or you could collect data from the last several acquisitions. An array of data from several acquisitions will likely mean little without some statistical analysis.

To get a clearer picture of this array of data, you would likely want to calculate some measure of central tendency. **A measure of central tendency is the central value around which data observations (e.g., historical prices) tend to cluste r.** It is the central value of the distribution. This section will examine calculation of the three most common and useful measures of central tendency: the **arithmetic mean**, the **median**, and the **mode**.

Calculating the Arithmetic Mean

The arithmetic mean (or simply the mean or average) is the measure of central tendency most commonly used in contract pricing. To calculate the mean, sum all observations in a set of data and divide by the total number of observations involved. The formula for this calculation is:

$$\bar{X} = \frac{\Sigma X}{n}$$

Where:

$\bar{X}$	=	sample mean
Σ	=	summation of all the variables that follow the symbol (e.g., ΣX represents the sum of all X values)
X	=	the value of the observation of the variable X
n	=	total number of observations in the sample

3.2 Measuring Central Tendency (cont)

Calculating the Arithmetic Mean (cont)

For Example. Suppose that a contract specialist is trying to determine the production lead time (PLT) of an electronic component and she has the following historic sample data:

Item	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
PLT (Months)	9	7	9	9	11	8	11	8

Though it is not required for the computation of the mean we will first array the data from smallest to largest in order to facilitate the computation of two other measures of central tendency, the median and the mode:

X	7	8	8	9	9	9	11	11

$$\bar{X} = \frac{\Sigma X}{n}$$

$$\bar{X} = \frac{7 + 8 + 8 + 9 + 9 + 9 + 11 + 11}{8} = \frac{72}{8} = 9 \text{ months}$$

Calculating the Median

The median is a measure of central tendency that is often used when a few observations might pull the measure from the center of the remaining data. For example, average housing value in an area is commonly calculated using the median, because a few extremely high-priced homes could result in a mean that presents an overvalued picture of the average home price. The median is the **middle value of a data set when the observations are arrayed from the lowest to the highest** (or from the highest to the lowest). If the data set contains an even number of observations, the median is the arithmetic mean of the two middle observations. In the production lead time example above, there is an even number of data points; so you would calculate the median as:

$$Md = \frac{9 + 9}{2} = 9 \text{ months}$$

3.2 Measuring Central Tendency (cont)

Calculating the Mode	Occasionally, you may only want to know which value occurs most often. The mode is **the value of the observation that occurs most often in the data set** (i.e. the value with the highest frequency). In the production lead time example above, the mode is nine. Nine occurs three times, once more than any other value. Since this data set has only one mode, the distribution is called **unimodal**. A distribution containing **two values occurring an equal number of times is called bimodal. One with more than two values occurring an equal number of times is called multimodal**.

3.3 Measuring Dispersion

Measures of Dispersion

As described in the previous section, the mean is the measure of central tendency most commonly used in contract pricing. Though the mean for a data set is a value around which the other values tend to cluster, it conveys no indication of the closeness of the clustering (that is, the dispersion). All observations could be close to the mean or far away

If you want an indication of how closely these other values are clustered around the mean, you must look beyond measures of central tendency to measures of dispersion. This chapter will examine:

- Several **measures of absolute dispersion** commonly used to describe the variation within a data set -- the range, mean absolute deviation, variance, and standard deviation.
- One **measure of relative dispersion** -- the coefficient of variation.

Example of Differences in Dispersion

Assume that you have the following scrap rate data for two contractor departments:

	Contractor Scrap Rate Data	
Month	**Dept. A: Fabrication**	**Dept. B: Assembly**
February	.065	.050
March	.035	.048
April	.042	.052
May	.058	.053
June	.032	.048
July	.068	.049
Total	.300	.300
Mean	.050	.050

The mean for both departments is the same -- 5 percent. However, the monthly scrap rates in Department B show less variation (dispersion) around the mean. As a result, you would probably feel more comfortable forecasting a scrap rate of 5 percent for Department B than you would for Department A.

Differences in dispersion will not always be so obvious. The remainder of this section will demonstrate how you can quantify dispersion using the five measures identified above.

3.3 Measuring Dispersion (cont)

Calculating the Range

Probably the quickest and easiest measure of dispersion to calculate is the range. The range (R) of a set of data is **the difference between the highest (H) and lowest (L) observations**. The higher the range, the greater the amount of variation in a data set.

$$R = H - L$$

For example: By comparing the range for Departments A scrap-rate data with the range for Department B, you can easily determine that the historical data from Department A shows wider dispersion than the data from Department B.

$$R_{DEPT\ A} = .068 - .032 = .036$$

$$R_{DEPT\ B} = .053 - .048 = .005$$

Steps for Calculating the Mean Absolute Deviation

The mean absolute deviation (MAD) is the average absolute difference between the observed values and the arithmetic mean (average) for all values in the data set.

If you subtracted the mean from each observation, some answers would be positive and some negative. The sum of all the deviations (differences) will always be zero. That tells you nothing about how far the average observation is from the mean. To make that computation, you must use the absolute difference between each observation and the mean. An absolute difference is the difference without consideration of sign, and all absolute values are written as positive numbers. For example: If the average is eight and the observed value is 10, the calculated difference is two (10 - 8 = 2), and the absolute difference is also two. If the average is eight and the observed value is six, the calculated difference is a negative two (6 - 8 = -2), but the absolute difference would be two (without the negative sign). **Note: In equations, absolute values are identified using a vertical line before and after the value (e.g., |X| identifies the absolute value of X).**

3.3 Measuring Dispersion (cont)

Steps for Calculating the Mean Absolute Deviation (cont)

To compute the MAD, use the following 5-step process:

Step 1. Calculate the arithmetic mean of the data set.

Step 2. Calculate the deviation (difference) between each observation and the mean of the data set.

Step 3. Convert each deviation to its absolute value (i.e., its value without considering the sign of the deviation).

Step 4. Sum the absolute deviations.

Step 5. Divide the total absolute deviation by the number of observations in the data set.

$$\text{MAD} = \frac{\Sigma|X - \bar{X}|}{n}$$

Calculating the Mean Absolute Deviations for the Scrap-Rate Example

We can use the 5-step process described above to calculate the scrap-rate MAD values for Departments A and B of the scrap-rate example.

Calculate the MAD for Department A:

Step 1. Calculate the arithmetic mean of the data set.

We have already calculated the mean rate for Department A of the scrap-rate example -- .05.

Step 2. Calculate the deviation (difference) between each observation and the mean of the data set $(X - \bar{X})$.

DEPARTMENT A (FABRICATION)		
X	$\bar{X}$	$X - \bar{X}$
.065	.050	.015
.035	.050	-.015
.042	.050	-.008
.058	.050	.008
.032	.050	-.018
.068	.050	.018

3.3 Measuring Dispersion (cont)

Calculating the Mean Absolute

Step 3. Convert each deviation to its absolute value ($|X - \bar{X}|$).

Deviations for the Scrap-Rate Example (cont)

DEPARTMENT A (FABRICATION)			
X	$\bar{X}$	$X - \bar{X}$	$\lvert X - \bar{X} \rvert$
.065	.050	.015	.015
.035	.050	-.015	.015
.042	.050	-.008	.008
.058	.050	.008	.008
.032	.050	-.018	.018
.068	.050	.018	.018

Step 4. Sum the absolute deviations ($\Sigma|X - \bar{X}|$).

DEPARTMENT A (FABRICATION)			
X	$\bar{X}$	$X - \bar{X}$	$\lvert X - \bar{X} \rvert$
.065	.050	.015	.015
.035	.050	-.015	.015
.042	.050	-.008	.008
.058	.050	.008	.008
.032	.050	-.018	.018
.068	.050	.018	.018
		Total	.082

Step 5. Divide the total absolute deviation by the number of observations in the data set.

$$MAD = \frac{\Sigma |X - \bar{X}|}{n}$$

$$MAD_{DEPT A} = \frac{.082}{6} = .014$$

3.3 Measuring Dispersion (cont)

Calculating the Mean Absolute Deviations for the Scrap-Rate Example (cont)

Calculate the MAD for Department B

Step 1. Calculate the arithmetic mean of the data set.

We have also calculated the mean rate for Department B of the scrap-rate example -- .05.

Steps 2 - 4. Calculate the deviation between each observation and the mean of the data set; convert the deviation to its absolute value; and sum the absolute deviations.

The following table demonstrates the three steps required to calculate the total absolute deviation for Department B:

DEPARTMENT A (FABRICATION)			
X	$\bar{X}$	$X - \bar{X}$	$\lvert X - \bar{X} \rvert$
.050	.050	.000	.000
.048	.050	-.002	.002
.052	.050	.002	.002
.053	.050	.003	.003
.048	.050	-.002	.002
.049	.050	-.001	.001
		Total	.010

Step 5. Divide the total absolute deviation by the number of observations in the data set.

$$\text{MAD} = \frac{\Sigma \mid X - \bar{X} \mid}{n}$$

$$\text{MAD}_{\text{DEPTB}} = \frac{.010}{6} = .002$$

Compare MAD values for Department A and Department B:

The MAD for Department A is .014; the MAD for Department B is .002. Note that the MAD for Department B is much smaller than the MAD for Department A. This comparison once again confirms that there is less dispersion in the observations from Department B.

3.3 Measuring Dispersion (cont)

Steps for Calculating the Variance

Variance (S^2) is one of the two most popular measures of dispersion (the other is the standard deviation which is described below). The variance of a sample is the average of the squared deviations between each observation and the mean.

However, statisticians have determined when you have a relatively small sample, you can get a better estimate of the true population variance if you calculate variance by dividing the sum of the squared deviations by n - 1, instead of n.

$$S^2 = \frac{\Sigma\left(X - \overline{X}\right)^2}{n - 1}$$

The term, n - 1, is known as the number of **degrees of freedom** that can be used to estimate population variance.

This adjustment is necessary, because samples are usually more alike than the populations from which they are taken. Without this adjustment, the **sample variance is likely to underestimate the true variation in the population**. Division by n – 1 in a sense artificially inflates the sample variance but in so doing, it makes the sample variance a better estimator of the population variance. As the sample size increases, the relative affect of this adjustment decreases (e.g., dividing by four rather than five will have a greater affect on the quotient than dividing by 29 instead of 30).

To compute the variance, use this 5-step process:

Step 1. Calculate the arithmetic mean of the data set.

Step 2. Calculate the deviation (difference) between each observation and the mean of the data set.

Step 3. Square each deviation.

Step 4. Sum the squared deviations.

Step 5. Divide the sum of the squared deviations by n-1.

$$S^2 = \frac{\Sigma\left(X - \overline{X}\right)^2}{n - 1}$$

3.3 Measuring Dispersion (cont)

Calculating the Variances for the Scrap-Rate Example

We can demonstrate the 5-step process described above to calculate the scrap-rate variances for Departments A and B of the scrap-rate example.

Calculate the variance for Department A:

Step 1. Calculate the arithmetic mean of the data set.

We have already calculated the mean rate for Department A of the scrap-rate example -- .05.

Step 2. Calculate the deviation (difference) between each observation and the mean of the data set $(X - \bar{X})$.

The deviations for Department A are the same as we calculated in calculating the mean absolute deviation.

DEPARTMENT A (FABRICATION)		
X	**$\bar{X}$**	**$X - \bar{X}$**
.065	.050	.015
.035	.050	-.015
.042	.050	-.008
.058	.050	.008
.032	.050	-.018
.068	.050	.018

Step 3. Square each deviation $X - \bar{X}$.

DEPARTMENT A (FABRICATION)			
X	**$\bar{X}$**	**$X - \bar{X}$**	**$(X - \bar{X})^2$**
.065	.050	.015	.000225
.035	.050	-.015	.000225
.042	.050	-.008	.000064
.058	.050	.008	.000064
.032	.050	-.018	.000324
.068	.050	.018	.000324

3.3 Measuring Dispersion (cont)

Calculating the Variances for the Scrap- Rate Example (cont)

Step 4. Sum the total absolute deviations $\left[\Sigma(X-\bar{X})^2\right]$.

DEPARTMENT A (FABRICATION)			
X	$\bar{X}$	**X – $\bar{X}$**	$\Sigma(X-\bar{X})^2$
.065	.050	.015	.000225
.035	.050	-.015	.000225
.042	.050	-.008	.000064
.058	.050	.008	.000064
.032	.050	-.018	.000324
.068	.050	.018	.000324
		Total	.001226

Step 5. Divide the sum of the squared deviations by n-1.

$$S^2 = \frac{(X-\bar{X})^2}{n-1}$$

$$S^2_{\text{Dept A}} = \frac{.001226}{6-1} = \frac{.001226}{5} = .000245$$

3.3 Measuring Dispersion (cont)

Calculating the Mean Absolute Deviations for the Scrap-Rate Example

Calculate the variance for Department B

Step 1. Calculate the arithmetic mean of the data set.

We have also calculated the mean rate for Department B of the scrap-rate example -- .05.

Steps 2 - 4. Calculate the deviation between each observation and the mean of the data set; convert the deviation to its absolute value; and sum the absolute deviations.

The following table demonstrates the three steps required to calculate the total absolute deviation for Department B:

DEPARTMENT B (FABRICATION)			
X	$\bar{X}$	$X - \bar{X}$	$\Sigma(X - \bar{X})^2$
.050	.050	.000	.000000
.048	.050	-.002	.000004
.052	.050	.002	.000004
.053	.050	.003	.000009
.048	.050	-.002	.000004
.049	.050	-.001	.000001
		Total	.000022

Step 5. Divide the sum of the squared deviations by n-1.

$$S^2 = \frac{(X - \bar{X})^2}{n - 1}$$

$$S^2_{\text{Dept B}} = \frac{.000022}{6 - 1} = \frac{.000022}{5} = .000004$$

Compare variances for Department A and Department B:

The variance for Department A is .000245; thevariance for Department B is .000004. Once again, the variance comparison confirms that there is less dispersion in the observations from Department B.

3.3 Measuring Dispersion (cont)

Concerns About Using the Variance as a Measure of Dispersion

There are two concerns commonly raised about using the variance as a measure of dispersion:

- As the deviations between the observations and the mean grow, the variation grows much faster, because all the deviations are squared in variance calculation.
- The variance is in a different denomination than the values of the data set. For example, if the basic values are measured in feet, the variance is measured in square feet; if the basic values are measured in terms of dollars, the variance is measured in terms of "square dollars."

Calculating the Standard Deviation

You can eliminate those two common concerns by using the standard deviation (S) -- the square root of the variance.

$$S = \sqrt{S^2}$$

For example: You can calculate the standard deviation for Departments A and B of the scrap-rate example:

$$S_{DEPTA} = \sqrt{.000245} = .015652$$

$$S_{DEPTB} = \sqrt{.000004} = .002000$$

NOTE: Both the variance and the standard deviation give increasing weight to observations that are further away from the mean. Because all values are squared, a single observation that is far from the mean can substantially affect both the variance and the standard deviation.

3.3 Measuring Dispersion (cont)

Empirical Rule

The standard deviation has one characteristic that makes it extremely valuable in statistical analysis. In a distribution of observations that is approximately symmetrical (normal):

- The interval $\overline{X} \pm 1S$ includes approximately 68 percent of the total observations in the population.
- The interval $\overline{X} \pm 2S$ includes approximately 95 percent of the total observations in the population.
- The interval $\overline{X} \pm 3S$ includes approximately 99.7 percent of the total observations in the population.

This relationship is actually a finding based on analysis of the normal distribution (bell shaped curve) that will be presented later in the chapter.

Coefficient of Variation

Thus far we have only compared two samples with equal means. In that situation, the smaller the standard deviation, the smaller the relative dispersion in the sample observations. However, that is not necessarily true when the means of two samples are not equal.

If the means are not equal, you need a measure of relative dispersion. The coefficient of variation (CV) is such a measure.

$$CV = \frac{S}{\overline{X}}$$

For example: Which of the following samples has more relative variation?

$$\text{Sample C}: \quad \overline{X} = 25 \qquad S = 5$$

$$\text{Sample D}: \quad \overline{X} = 100 \qquad S = 10$$

Calculate CV for Sample C:

$$CV_C = \frac{5}{25} = .20$$

Calculate CV for Sample D:

$$CV_D = \frac{10}{100} = .10$$

3.3 Measuring Dispersion (cont)

Coefficient of Variation (cont)

Compare the two CV values:

Even though the standard deviation for Sample D is twice as large as the standard deviation for Sample C, the CV values demonstrate that Sample D exhibits less relative variation. This is true because the mean for Sample D is so much larger than the mean for Sample C.

Note: We could calculate CV for the scrap-rate example, but such a calculation is not necessary because the means of the two samples are equal.

3.4 Establishing a Confidence Interval

Confidence Interval

Each time you take a sample from a population of values you can calculate a mean and a standard deviation (S). Even if all the samples are the same size and taken using the same random procedures, it is unlikely that every sample will have the same mean and standard deviation. However, if you could collect all possible samples from the normally distributed population and calculate the mean value for all the sample means, the result would be equal to the population mean. In statistical terminology, the mean of the sampling distribution is equal to the population mean.

You can combine the sample mean and sample standard deviation with an understanding of the shape of distribution of sample means to develop a **confidence interval -- a probability statement about an interval which is likely to contain the true population mean.**

> **For example:** Suppose that you are preparing a solicitation for an indefinite-quantity transmission overhaul contract to support a fleet of 300 light utility trucks. You believe that you can develop an accurate estimate of the number of transmissions that will require a major overhaul during the contract period, if you can determine the date of the last major overhaul for each vehicle transmission and estimate the period between overhauls. You select a simple random sample (without replacement) of 25 vehicle maintenance records from the 300 fleet vehicle maintenance records. Analyzing the sample, you find that the mean time between overhauls is 38 months and the sample standard deviation is 4 months. Based on this analysis, your point estimate of the average transmission life for all vehicles in the fleet (the population mean) would be 38 months. But you want to establish reasonable estimates of the minimum and maximum number of repairs that will be required during the contract period. You want to be able to state that you are 90% confident that the average fleet transmission life is within a defined range (e.g., between 36 and 40 months).

To make this type of statement, what you need to do is establish a confidence interval. You can establish a confidence interval using the sample mean, the standard error of the mean, and an understanding of the normal probability distribution and the t distribution.

3.4 Establishing a Confidence Interval (cont)

Calculating the Standard Error of the Mean

If the population is normally distributed, the standard error of the mean is equal to the population standard deviation divided by the square root of sample size. Since we normally do not know the population standard deviation, **we normally use the sample standard deviation to estimate the population standard deviation.** Using the sample standard deviation, you can calculate the standard error of the mean $(S_{\bar{x}})$ using the following equation:

$$S_{\bar{x}} = \frac{S}{\sqrt{n}}$$

Though the population mean and the population standard deviation are not normally known, we assume that cost or pricing data are normally distributed. This is a critical assumption because it allows us to construct confidence intervals (negotiation ranges) around point estimates (Government objectives).

Calculate the Standard Error of the Mean for the Transmission Example.

Remember that: S = 4 months

n = 25 vehicle maintenance records

$$S_{\bar{x}} = \frac{S}{\sqrt{n}}$$

$$S_{\bar{x}} = \frac{4}{\sqrt{25}}$$

$$S_{\bar{x}} = \frac{4}{5}$$

$$S_{\bar{x}} = .8 \text{ months}$$

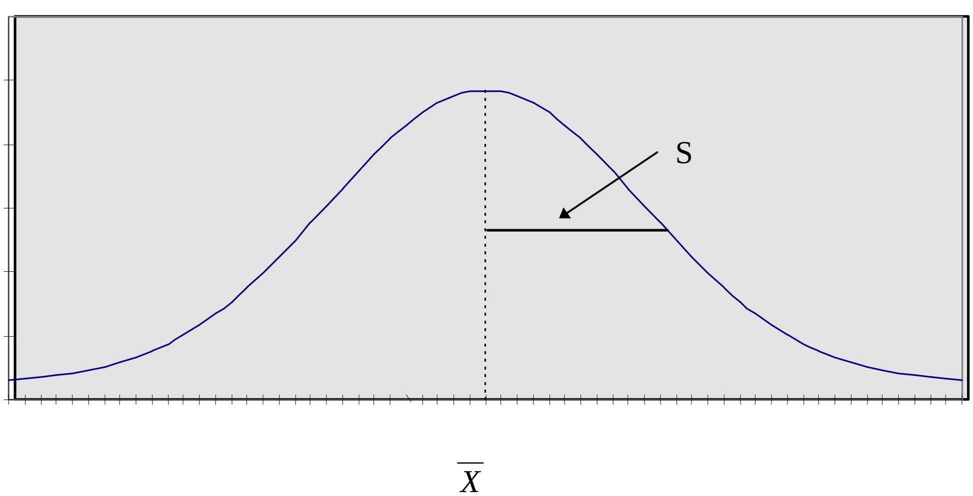
3.4 Establishing a Confidence Interval (cont)

Normal Probability Distribution

The normal probability distribution is the **most commonly used continuous distribution**. Because of its unique properties, **it applies to many situations in which it is necessary to make inferences about a population by taking samples**. It is a close approximation of the distribution of such things as human characteristics (e.g., height, weight, and intelligence) and the output of manufacturing processes (e.g., fabrication and assembly). The normal probability distribution provides the probability of a continuous random variable, and has the following characteristics:

- It is a symmetrical (i.e., the mean, median, and mode are all equal) distribution; half of the possible outcomes are on each side of the mean.
- The total area under the normal curve is equal to 1.00. In other words, there is a 100 percent probability that the possible observations drawn from the population will be covered by the normal curve.
- It is an asymptotic distribution (the tails approach the horizontal axis but never touch it).
- It is represented by a smooth, unimodal, bell-shaped curve, usually called a "normal probability density function" or "normal curve."
- It can be defined by two characteristics—the mean and the standard deviation. (See the figure below.)

NORMAL CURVE



3.4 Establishing a Confidence Interval (cont)

Conditions Necessary for Using the Normal Distribution

Use the normal curve to construct confidence intervals around a sample mean when the following condition is met:

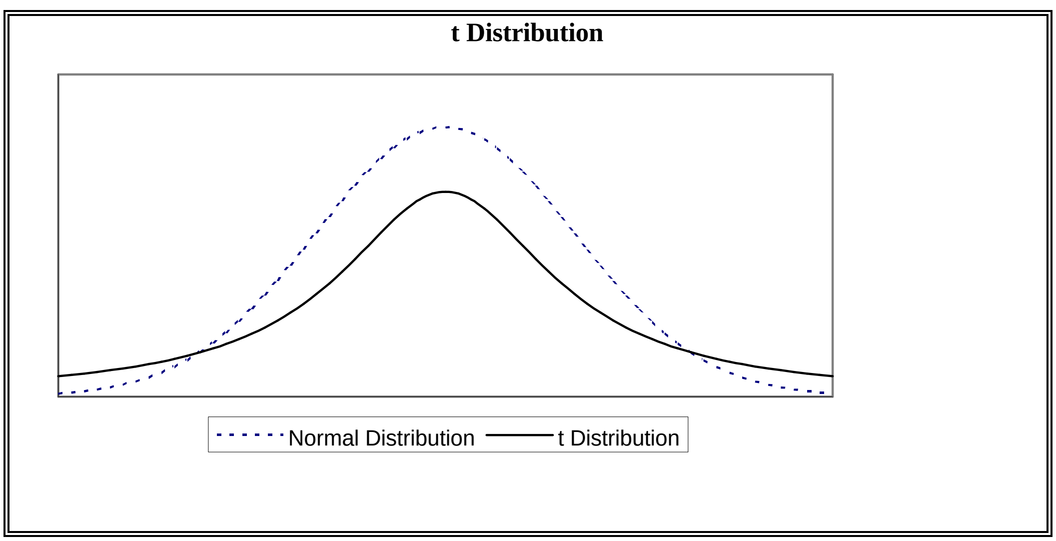
- The population mean and standard deviation are known.

t Distribution

In contract pricing, the conditions for using the normal curve are rarely met. As a result, you will normally need to use a variation of the normal distribution called the "**t-distribution**."

The t distribution has the following characteristics:

- It is symmetrical, like the normal distribution, but it is a flatter distribution (higher in the tails).
- Whereas a normal distribution is defined by the mean and the standard deviation, the t distribution is defined by degrees of freedom.
- There is a different t distribution for each sample size.
- As the sample size increases, the shape of the t distribution approaches the shape of the normal distribution.



3.4 Establishing a Confidence Interval (cont)

Relationship Between Confidence Level and Significance Level

Confidence Level. The term confidence level refers to the confidence that you have that a particular interval includes the population mean. In general terms, a confidence interval for a population mean when the population standard deviation is unknown and n $\leq$ 30 can be constructed as follows:

$$\bar{X} \pm tS_{\bar{X}}$$

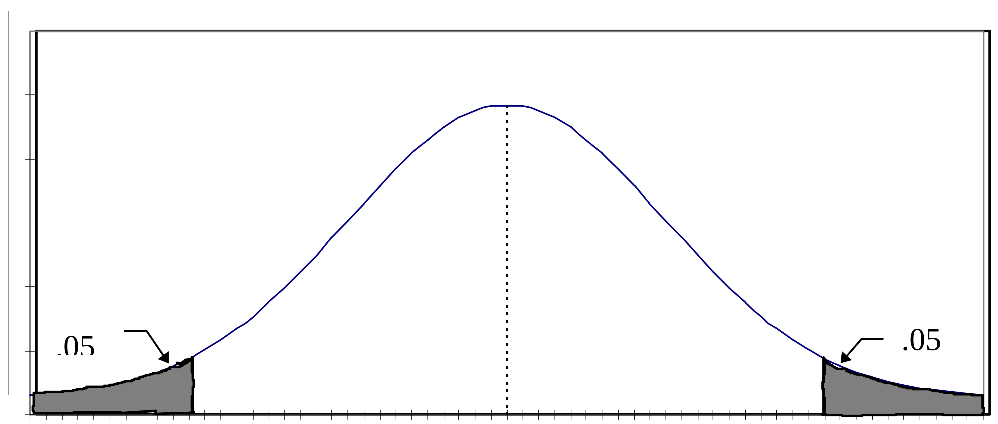
Where:

t = t Table value based on sample size and the significance level

$S_{\bar{X}}$ = Standard error of the mean

Significance Level. The significance level (α) is **equal to 1.00 minus the confidence level**. For example if the confidence level is 95 percent, the significance level is 5 percent; if the confidence level is 90 percent, the significance level is 10 percent. The significance level is, then, **the area outside the interval which is likely to contain the population mean**.

The figure below depicts a 90 percent confidence interval. Note that there is a 5 percent risk that the population mean is greater than the confidence interval and a 5 percent risk that the mean is less than the confidence interval.



90 Percent Confidence Interval

3.4 Establishing a Confidence Interval (cont)

Setting the Significance Level

When you set the significance level, you must determine the amount of risk you are willing to accept that the confidence interval does not include the true population mean. As the amount risk that you are willing to accept decreases, the confidence interval will increase. In other words, to be more sure that the true population mean in included in the interval, you must widen the interval.

Your tolerance for risk may vary from situation to situation, but for most pricing decisions, a significance level of .10 is appropriate.

Steps in Obtaining a t Value for Confidence Interval Construction

After you have taken a random sample, calculated the sample mean and the standard error of the mean, you need only a value of t to construct a confidence interval. To obtain the appropriate t value, use the following steps:

Step 1. **Determine your desired significance level.** As stated above, for most contract pricing situations, you will find a significance level of .10 appropriate. That will provide a confidence level of .90 (1.00 - .10 = .90).

Step 2. **Determine the degrees of freedom.** Degrees of freedom are the sample size minus one (n - 1).

Step 3. **Determine the t value from the t Table**. Find the t value at the **intersection of the df row and the .10 column**.

Constructing a Confidence Interval for the Transmission Overhaul Example

Recall the transmission overhaul example, where you wanted to estimate the useful life of transmissions of a fleet of 300 light utility trucks. We took a random sample of size n = 25 and calculated the following:

$$\bar{X} \pm tS_{\bar{X}}$$

Where:

$\bar{X}$	= 38 months
S	= 4 months
$S_{\bar{X}}$	= .8 months

3.4 Establishing a Confidence Interval (cont)

Constructing a Confidence Interval for the Transmission Overhaul Example (cont)

Assume that you want to construct a 90% confidence interval for the population mean (the actual average useful life of the transmissions). You have all the values that you need to substitute into the formula for confidence interval **except the t value**. To determine the t value for a confidence interval, use the following steps:

Calculate t Value.

Step 1. Determine the significance level.

Use the significance level is .10.

Step 2. Determine the degrees of freedom.

df = n - 1

df = 25 – 1

df = 24

Step 3. Determine the t value from the t Table.

Find the t value at the **intersection of the df = 24 row and the .10 column**. The following table is an excerpt of the t Table (see Appendix B for a complete t Table):

PARTIAL t TABLE	
df	**t**
- - - - - - - - - - -	
23	1.714
24	1.711
25	1.708
26	1.706
- - - - - - - - - - -	

Reading from the table, **the appropriate t value is 1.711.**

3.4 Establishing a Confidence Interval (cont)

Constructing a Confidence Interval for the Transmission Overhaul Example (cont)

Use t Value and Other Data to Construct Confidence Interval.

The confidence interval for the true population mean (the actual average useful life of the transmissions) would be:

$$\bar{X} \pm tS_{\bar{X}}$$

$$38 \pm 1.711(.8)$$

$$38 \pm 1.37$$

Confidence interval for the population mean (μ): **36.63 $\leq$ μ $\leq$ 39.37**

That is, **you would be 90 percent confident that the average useful life of the transmissions is between 36.63 and 39.37 months** .

3.5 Using Stratified Sampling

Stratified Sampling Applications in Contract Pricing

You should consider using sampling when you have a large amount of data and limited time to conduct your analysis. Using stratified sampling allows you to concentrate your efforts on the items with the greatest potential for cost reduction while using random sampling procedures for to identify any no general pattern of overpricing of smaller value items.

The most common contract pricing use of stratified sampling is analysis of detailed material cost proposals. Often hundreds, even thousands, of material items are purchased to support production of items and systems to meet Government requirements. To analyze the quantity requirements and unit prices for each item would be extremely time consuming and expensive. Because often more than 50 percent of the contract price is in material items, effective review is essential. The overall environment is custom made for the use of stratified sampling.

Steps in Stratified Sampling

While there are many different methods of sampling, stratified sampling is usually the most efficient and effective method of sampling for cost analysis. The components of the proposal cost (e.g., a bill of materials) to be analyzed are divided into two or more groups or strata for analysis. One group or stratum is typically identified for 100 percent review and the remaining strata are analyzed on a sample basis.

3.5 Using Stratified Sampling (cont)

Steps in Stratified Sampling (cont)

Analyze and develop a prenegotiation position for each item in this stratum. Total those positions to arrive at an overall prenegotiation position for the stratum.

Step 2. Group the remaining items into one or more stratum for analysis.

The number of additional strata necessary for analysis will depend on several factors:

- If the remaining items are relatively similar in price and other characteristics (e.g., industry, type of source, type of product), only one additional stratum may be required.
- If the remaining items are substantially different in price or other characteristics, more than one stratum may be required. For example, you might create one stratum for items with a total price of $5,001 to $20,000 and another stratum for all items with a total price of $5,000 or less.
- If you use a sampling procedure that increases the probability of selecting larger dollar items (such as the dollar unit sampling procedure available in E-Z-Quant), the need for more than one stratum may be reduced.

Step 3. Determine the number of items to b e sampled in each stratum.

You must analyze all items in the strata requiring 100 percent analysis. For all other strata, you must determine how many items you will sample. You should consider several factors in determining sample size. The primary ones are variability, desired confidence, and the total count of items in the stratum. Use statistical tables or computer programs to determine the proper sample size for each stratum.

Step 4. Select items for analysis.

In the strata requiring random sampling, each item in the stratum must have an equal chance of being selected and each item must only be selected once for analysis. Assign each item in the population a sequential number (e.g., 1, 2, 3; or 1001, 1002, 1003. Use a table of random numbers or computer generated random numbers to identify the item numbers to be included in the sample.

(continued on next page)

3.5 Using Stratified Sampling (cont)

Steps in Stratified Sampling (cont)

Step 5. Analyze all items identified for analysis summing recommended costs or prices for the 100 percent analysis stratum and developing a "decrement factor" for any stratum being randomly sampled.

In the stratum requiring 100 percent analysis, you can apply any recommended price reductions directly to the items involved. In any stratum where you use random sampling, you must apply any recommended price reductions to all items in the stratum.

- Analyze the proposed cost or price of each sampled item.
- Develop a "should pay" cost or price for the item.

  You must do this for every item in the sample, regardless of difficulty, to provide statistical integrity to the results. If you cannot develop a position on a sampled item because offeror data for the item is plagued by excessive misrepresentations or errors, you might have to discontinue your analysis and return the proposal to the offeror for correction and update.
- Determine the **average** percentage by which should pay prices for the sampled items differ from proposed prices. This percentage is the **decrement factor**. [1]
- Calculate the confidence interval for the decrement factor.

Step 6. Apply the decrement factor to the total proposed cost of all items in the stratum.

The resulting dollar figure is your prenegotiation position for the stratum. Similarly, use confidence intervals to develop the negotiation range.

Step 7. Total prenegotiation positions for all strata to establish your overall position on the cost category.

[1] There are a number of techniques for determining the “average” percentage which will produce different results. For example, you could (1) determine the percentage by which each should pay price differs from each proposed price, (2) sum the percentages, and (3) divide by the total number of items in the sample. This technique gives equal weight to all sampled items in establishing the decrement factor. Or you could (1) total proposed prices for all sampled items, (2) total the dollar differences between should pay and proposed prices, and (3) divide the latter total by the former total. This technique gives more weight to the higher priced sampled items in establishing the decrement factor.

3.5 Using Stratified Sampling (cont)

Stratified Sampling Example

Assume you must analyze a cost estimate that includes 1,000 material line items with a total cost of $2,494,500. You calculate that you must analyze a simple random sample 50 line items.

Step 1. Identify a stratum of items that merit "100 percent analysis."

You want to identify items that merit 100 percent analysis because of their relatively high cost. To do this, you prepare a list the 1,000 line items organized from highest extended cost to lowest extended. The top six items on this list look like this:

Item 1	$675,000
Item 2	$546,500
Item 3	$274,200
Item 4	$156,800
Item 5	$101,300
Item 6	$ 26,705

Note that the top five items $1,753,800 (about 70 percent of the total material cost). You will commonly find that a few items account for a large portion of proposed material cost. Also note that there is a major drop from $101,300 to $26,705. This is also common. Normally, you should look for such break points in planning for analysis. By analyzing Items 1 to 5, you will consider 70 percent of proposed contract cost. You can use random sampling procedures to identify pricing trends in the remaining 30 percent.

Step 2. Group the remaining items into one or more stratum for analysis.

A single random sampling stratum is normally adequate unless there is very broad range of prices requiring analysis. This typically only occurs with multimillion dollar proposals. Here, the extended prices for the items identified for random sampling range from $5.00 to $26,705. While this is a wide range, the dollars involved seem to indicate that a single random sampling strata will be adequate.

Step 3. Determine the number of items to be sampled in each stratum.

Based on the dollars and the time available, you determine to sample a total of 20 items from the remaining 995 on the bill of materials.

3.5 Using Stratified Sampling (cont)

Stratified Sampling (cont)

Step 4. Select items for analysis.

One way that you could select items for analysis would be putting 1,000 sheets of paper, one for each line item, into a large vat, mix them thoroughly, and select 20 slips of paper from the vat. If the slips of paper were thoroughly mixed, you would identify a simple random sample.

A less cumbersome method would be to use a random number table (such as the example on the next page) to pick a simple random sample. A random number table is one in which the digits 0 through 9 appear in no particular pattern and each digit has an equal probability (1/10) of occurring. The number of digits in each random number should be greater than or equal to the number of digits we have assigned to any element in the population.

To sample a population of 995 items, numbered 1 to 995, random numbers must have at least three digits. Since you are dealing with three digit numbers, we need only use the first three digits in each random number on the next page. Though you can start at any point, it is customary to select a start point at random. Assume that you start at Row 2, Column 3. The first number is 365; hence the first line item in our sample would be the item identified as 365, the second 265, and the third 570 etc. Proceed sequentially until all 20 sample line items have been selected. When you get to the end of the table you would go to Row 1, Column 4.

Step 5. Analyze all items identified for analysis summing recommended costs or prices for the 100 percent analysis stratum and developing a "decrement factor" for any stratum being randomly sampled.

Results of 100 Percent Analysis. Use of the 100 percent analysis is straight forward. In this example, the offeror proposed a total of $1,753,800 for 5 line items of material. An analysis of these items found that the unit cost estimates were based on smaller quantities than required for the contract. When the full requirement was used, the total cost for those five items decreased to $1,648,600. Since the analysis considered all items in the stratum, you simply need to use the findings in objective development.

Random Sample Results. The random sample included 20 items with an estimated cost of $75,000. Analysis finds that the cost of the sampled items should be only 98 percent of the amount proposed. However, the confidence interval indicates that costs may range from 96 to 100 percent of the costs proposed.

3.5 Using Stratified Sampling (cont)

Stratified
Sampling (cont)

RANDOM NUMBER TABLE

6698450	3756022	1379873	1091250	2464369
8230671	2261081	3651245	2403835	3796196
3307349	1606742	2651096	8733169	8020931
4557294	4487241	5709649	8443898	4186082
4867520	4593304	7859673	4715519	8672562
5256666	7532438	4932550	3189230	5767320
7629754	4443063	2645070	5402375	6320447
7388907	6027106	4214053	7360106	6648860
6976195	3827223	2887476	3195410	3200243
1120094	8270134	2226093	8064010	2436229
7471111	8911616	1633582	7834674	5557861
7569916	6576524	4641095	6893288	3204198
1317932	2313736	3428616	2105629	8731495
3440584	6406297	5135877	4804056	2250677
6472532	3716016	5304620	7202428	6889909
2676982	8524653	6426810	2026354	6881881
1662752	7385363	1674986	6997446	8676962
7408077	8443370	1586219	5255558	3478729
5564193	3848937	4385234	2467063	2012393
6419575	7393468	5890046	4375027	3094506
5382237	6976185	7170044	5523693	5298851
1407827	7079107	7446749	2960020	3833765
2090333	1717118	1766538	7504068	8285177
3264983	2462752	2857463	4812842	4262484
1574530	1746847	2933809	7563765	7835932
6645401	4831573	6157406	4779259	4099476
6934253	8266361	1628707	4227992	1857184
1063302	1395104	1288533	2096146	2439038
5189339	7209557	7587864	7348618	6628536
6771677	7664148	2353445	8520402	5754865
7643316	8106527	8565980	1717450	4791406
5672442	2860934	3900542	1489100	3935202
5152933	1853083	3480353	3585772	5243559
2493145	5267758	2810982	5584309	6599457
2270414	5399702	7469248	7473486	6377260
6916635	3169397	8863103	3877732	5318753
6543891	8863730	8075579	1949660	7169566

3.5 Using Stratified Sampling (cont)

Stratified Sampling Example (cont)

Step 6. Apply the decrement factor to the total proposed cost of all items in the stratum.

Results of 100 Percent Analysis. There is no need to apply a decrement factor to these items because the recommended cost of $1,648,600 resulted from analysis of all items in this stratum.

Random Sample Results. The assumption is that the sample is representative of the entire population. If the sample is overpriced, the entire population of is similarly overpriced. As a result the recommended cost objective would be $725,886, or 98 percent of the proposed $740,700. However, the confidence interval would be $711,072 (96 percent of $740,700) to $740,700 (100 percent of $740,700).

Step 7. Total prenegotiation positions for all strata to establish your overall position on the cost category.

Point Estimate. The total point estimate results from the 100 percent and random sample analyses would be $2,374,486 ($1,648,600 + $725,886).

Confidence Interval: The confidence interval would run from $2,359,672 ($1,648,600 + $711,072) to $2,389,300 ($1,648,600 + $740,700). Note that the position on the stratum subject would not change.

3.6 Identifying Issues and Concerns

Questions to Consider in Analysis

As you perform price or cost analysis, consider the issues and concerns identified in this section, whenever you use statistical analysis.

- **Are the statistics representative of the current contract situation?**

  Whenever historical information is used to make an estimate of future contract performance costs, assure that the history is representative of the circumstances that the contractor will face during contract performance.

- **Have you considered the confidence interval in developing a range of reasonable costs?**

  Whenever sampling procedures are used, different samples will normally result in different estimates concerning contractor costs. Assure that you consider the confidence interval in making your projections of future costs. Remember that there is a range of reasonable costs and the confidence interval will assist you in better defining that range.

- **Is the confidence interval so large as to render the point estimate meaningless as a negotiation tool?**

  If the confidence interval is very large (relative to the point estimate), you should consider increasing the sample size or other means to reduce the risk involved.

- **Is your analysis, including any sample analysis, based on current accurate and complete information?**

  A perfect analysis of information that is not current accurate and complete will likely not provide the best possible estimates of future contract costs.

- **Do the items with questioned pricing have anything in common?**

  If items with questioned pricing are related, consider collecting them into a separate stratum for analysis. For example, you might find that a large number of pricing questions are related to quotes from a single subcontractor. Consider removing all items provided by that subcontractor from existing strata for separate analysis.

CHAPTER 4

Cost Estimating Relationships

Learning Objective

At the End of This Chapter	At the end of this chapter you will be able to: **Classroom Learning Objective 4/1** Correctly use cost estimating relationships in estimating and analyzing contract cost or price.

4.0 Chapter Introduction

In This Chapter

In this chapter, you will learn to use cost estimating relationships to estimate and analyze estimates of contract cost and price.

Cost Estimating Relationship Definition

As the name implies, a cost estimating relationship (CER) is a technique used to estimate a particular cost or price by using an established relationship with an independent variable. If you can identify an independent variable (driver) that demonstrates a measurable relationship with contract cost or prices exist, you can develop a CER. A CER may be mathematically simple in nature (e.g., a simple ratio) or it may involve a complex equation.

Steps for Developing a Cost Estimating Relationship

Strictly speaking a CER is not a quantitative technique. It is a framework for using appropriate quantitative techniques to quantify a relationship between an independent variable and contract cost or price. CER development is a 6-step procedure. Follow the six steps whenever you develop a CER. Whenever you evaluate a CER developed by someone else, determine whether the developer followed the six steps properly.

Step 1. Define the dependent variable (e.g. , cost dollars, hours, and so forth.)

Define what the CER will estimate. Will the CER be used to estimate price, cost dollars, labor hours, material cost, or some other measure of cost? Will the CER be used to estimate total product cost or estimate the cost of one or more components? The better the definition of the dependent variable, the easier it will be to gather comparable data for CER development.

4.0 Chapter Introduction (cont)

Steps for Developing a Cost Estimating Relationship (cont)

Step 2. Select independent variables to be tested for developing estimates of the dependent variable.

In selecting potential independent variables for CER development, draw on personnel experience, the experience of others, and published sources of information. When developing a CER for a new state-of-the-art item, it is especially important for you to consult experts experienced with the appropriate technology and production methods.

In selecting the independent variable, consider the following factors:

- Variables should be quantitatively measurable. Parameters, such as maintainability, are difficult to use in estimating because they are difficult to measure quantitatively.
- Data availability is also important. If historical data cannot be obtained, it will be impossible to analyze and use the variable as a predictive tool. For example, an independent variable, such as physical dimensions or parts count, would be of little value during the conceptual phase of system development when the values of the independent variables are not known. Be especially wary of any CER based on 2 or 3 data observations.
- If there is a choice between developing a CER based on performance or physical characteristics, performance characteristics are generally the better choice, because performance characteristics are usually known before design characteristics.

Step 3. Collect data concerning the relationship between the dependent and independent variables.

Collecting data is usually the most difficult and time-consuming element of CER development. It is essential that all data be checked and double checked to ensure that it is relevant, comparable, relatively free of unusual costs.

Step 4. Explore the relationship between the dependent and independent variables.

During this step, you must determine the strength of the relationship between the independent and dependent variables. This phase of CER development can involve a variety of analytical techniques from simple graphic analysis to complex mathematical analysis. Simple ratio analysis, moving averages, and linear regression are some of the more commonly used quantitative techniques used in analysis.

4.0 Chapter Introduction (cont)

Steps for Developing a Cost Estimating Relationship (cont)

Step 5. Select the relationship that best predicts the dependent variable.

After exploring a variety of relationships, you must select the one that can best be used in predicting the dependent variable. Normally, this will be the relationship that best predicts the values of the dependent variable. A high correlation (relationship) between a potential independent variable and the dependent variable often indicates that the independent variable will be a good predictive tool. However, you must assure that the value of the independent variable is available in order for you to make timely estimates. If it is not, you may need to consider other alternatives.

Step 6. Document your findings.

CER documentation is essential to permit others involved in the estimating process to trace the steps involved in developing the relationship. Documentation should involve the independent variables tested, the data gathered, sources of data, time period of the data, and any adjustments made to the data.

4.1 Identifying Situations for Use

Situations for Use

You can use a cost estimating relationship (CER) in any situation where you quantify one of the following:

- **A relationship between one or more product characteristics and contract cost or price.**

  A **product-to-cost relationship** uses product physical or performance characteristics to estimate cost or product price. The characteristic or characteristics selected for CER development are usually not the only ones driving cost, but the movement of cost has been found to be related to changes in these characteristics. The following table identifies several product characteristic that have been used in CER development:

PRODUCT	INDEPENDENT VARIABLE
Building Construction	Floor space, roof surface area, wall surface
Gears	Net weight, gross weight, horsepower, number of driving axles, loaded cruising speed
Trucks	Empty weight, gross weight, horsepower, number of driving axles, loaded cruising speed
Passenger Car	Curb weight, wheel base, passenger space, horsepower
Turbine Engine	Dry weight, maximum thrust, cruise thrust, specific fuel consumption, by-pass ratio, inlet temperature
Reciprocating Engine	Dry weight, piston displacement, compression ratio, horsepower
Sheet Metal	Net weight, percent of scrap, number of holes drilled, number of rivets placed, inches of welding, volume of envelope
Aircraft	Empty weight, speed, useful load, wing area, power, landing speed
Diesel Locomotive	Horsepower, weight, cruising speed, maximum load on standard grade at standard speed

4.1 Identifying Situations for Use (cont)

Situations for Use (cont)

- **A relationship between one or more elements of contract cost and another element of contract cost or price.**

  A **cost-to-cost relationship** uses one or more elements of contract cost to estimate cost or product price. If you can establish a relationship between different elements of cost (e.g., between senior engineering labor hours and engineering technician hours), you can use a CER to reduce your estimating or analysis effort while increasing accuracy. If you can establish a relationship between an element of cost and total price (e.g., between direct labor cost and total price), you can use that information to supplement price analysis, without requiring extensive cost information.

4.2 Identifying and Using Rules of Thumb

Identifying Rules of Thumb

As you perform your market analysis, you should be on the lookout for cost estimating rules of thumb that are commonly used in the product marketplace. For example, when we compare the prices of houses, we typically do so in price per square foot. Using this rule of thumb, we can compare the cost of different houses or the same house in different parts of the country. There may be ways to develop more accurate estimates, but this rule of thumb is widely accepted, relatively easy to calculate, and it provides reasonably accurate results for many purposes. The same statement can probably be made about most rules of thumb. You may be able to develop better cost estimating relationships, but given the time available and the dollars involved, rules of thumb provide useful tools for contract pricing.

Validate a Rule of Thumb Before Using

Like any CER, a rule of thumb can be based on another cost, performance characteristic, or physical characteristic of the item being priced. Unlike other CERs, rules of thumb typically have not been validated for use in specific estimating situations. Validation has come from acceptable results produced in a variety of situations over a number of years. Before you use a rule of thumb, consider the 6-step CER development process and ask the following questions:

- **Can the rule of thumb reasonably be used to estimate what you are trying to estimate (e.g., cost dollars, hours, or product price)?**
- **Are there other rules of thumb that can be used to estimate the same cost or price?**
- **Is the data required to use this rule of thumb readily available?**
- **Does the rule of thumb provide reasonably accurate results?**
- **If more than one rule of thumb is available, which one appears to produce the most accurate estimates?**
- **Have technical experts or other buyers documented the results obtained from using the rule of thumb?**

4.2 Identifying and Using Rules of Thumb (cont)

Example of Rule of Thumb Validation

You just received two offers for 500 laboratory tables, each table is 4' X 6' (24 square feet of surface area), with oak frame and legs. The work surface is a unique composite material developed to meet unique Government requirements. The low offer is $425, but that offer is $175 less than the second low offer. As a result, you are concerned that the price may be unreasonably low. You have no acquisition history for this item and there are no similar items on the commercial market. As a result, you have been looking for a CER that you can use in your pricing decision. Another buyer, who has acquired similar tables, tells you that he has used a rule of thumb in pricing similar tables -- $19 per square feet of surface area. You want to know the answers to the following questions before you use it in making your own pricing decisions.

- **Can the rule of thumb reasonably be used to estimate what you are trying to estimate (e.g., cost dollars, hours, or product price)?**

  The answer appears to be yes. The buyer who recommended the CER has used it successfully. Additional information shows that he learned of the CER from the scientists who developed the table-top material.

- **Are there other rules of thumb that can be used to estimate the same cost or price?**

  You have asked several “experts” and have been unable to identify any other rules of thumb for estimating the price of these unique tables.

- **Is the data required to use this rule of thumb readily available?**

  Yes, you already know the table surface area.

- **Does the rule of thumb provide reasonably accurate results?**

  You have identified four recent acquisitions of similar tables and recorded the following information comparing the estimates made using the rule of thumb and the actual prices paid:

Sq Ft	Estimate	Actual Price	Percentage Difference
15	$285	$310	+ 8.8%
18	$342	$335	- 2.0%
32	$608	$580	- 4.6%

4.2 Identifying and Using Rules of Thumb (cont)

Example of Rule of Thumb Validation (cont)

This sample size is too small to perform an effective statistical analysis, but you can still subjectively evaluate rule of thumb estimate accuracy. All estimates are within 8.8 percent of the actual price. For a rule of thumb, this appears reasonably accurate, especially since our evaluation did not consider other acquisition situation differences (e.g., the number of tables on each contract).

- **If more than one rule of thumb is available, which one appears to produce the most accurate estimates?**

  In this example, there is only one known rule of thumb to consider.
- **Have technical experts or other buyers documented the results obtained from using the rule of thumb?**

  In this case, the buyer documented every contract file when the rule of thumb was used. Such documentation is not only valuable in supporting the contracting officer's decision on price reasonableness, it provides valuable information to any contracting officer considering rule of thumb use in the future.

Example of Using a Rule of Thumb in Estimate Development

Once you have determined that a rule of thumb is acceptable for estimate development, you must apply it to the available data. Using this rule of thumb, your estimate would be \$456 (24 x \$19). That estimate is about 7.3 percent higher than the low offer. Based on the rule of thumb, the price does not seem unreasonable.

4.3 Developing and Using Estimating Factors

Section Introduction

An estimating rate or factor is a simple ratio, used to estimate cost or price. The rule of thumb used to develop table price estimates in the previous section is an example -- $19 per square foot. As the size of the table top increases, the price estimate increases in direct proportion. Most rules of thumb are simple factors. Many CERs developed by Government or industry are also simple factors. They are easy to develop, easy to understand, and in many cases quite accurate.

Development and use of estimating rates and factors involves two important implicit assumptions.

- There is no element of the cost or price being estimated that is not related to the independent variable (i.e., there is no "fixed cost" that is not associated with the independent variable).
- The relationship between the independent variable and the cost being estimated is linear.

If you believe that there are substantial costs that cannot be explained by the relationship or that the relationship is not linear, you should either try to develop an equation that better tracks the true relationship or limit your use of the estimating factor to the range of the data used in developing the factor.

Example of Estimating Factor Development

Assume that you are negotiating a guard service contract for your facility and you want to develop a CER to assist you in estimating a should-pay contract price. Development should follow the 6-step CER process described in the chapter Introduction.

Step 1. Define the dependent variable.

The objective is to develop an estimate of the price that the Government should expect to pay for this contract.

Step 2. Select independent variables to be tested for developing estimates of the dependent variable.

Logically, the major driver of price in a guard service contract is the wages paid the security guards manning the various posts identified in the contract.

4.3 Developing and Using Estimating Factors (cont)

Example of Estimating Factor Development (cont)

Step 3. Collect data concerning the relationship between the dependent and independent variables.

You have collected information on prices, minimum manning requirements, and service contract wage-rate determinations for the guard service contract at your facility for the last three years. The minimum manning requirement for the current contract totals 75,000 (Guard II) hours. The Service Contract Act (SCA) Wage Rate for the current year is $10.00 per hour. The Estimated Direct Labor Cost for each year (Column D) is calculated by multiplying Estimated Direct Labor Hours (Column B) by the Service Contract Act Minimum Wage Rate (Column C).

A	B	C	D	E
Year	**Estimated Direct Labor Hours**	**SCA Minimum Wage Rate**	**Estimated Direct Labor Cost**	**Contract Price**
1	87,600	$9.15	$801,540	$1,346,585
2	78,840	$9.45	$745,038	$1,244,215
3	70,040	$9.50	$665,380	$1,124,490
Current	75,000	$10.00	$750,000	

Step 4. Explore the relationship between the dependent and independent variables.

The following table demonstrates calculation of the Price to Direct Labor Cost Ratio. The ratio (Column F) is calculated by dividing the contract price (Column E) by the estimated direct labor cost (Column D). In Year 1 for example, price was 1.68 times the Estimated Direct Labor Cost.

A	B	C	D	E	F
Year	**Estimated Direct Labor Hours**	**SCA Minimum Labor Rate**	**Estimated Direct Labor Cost**	**Contract Price**	**Price to Direct Labor Cost Ratio**
1	87,600	$9.15	$801,540	$1,346,585	1.68
2	78,840	$9.45	$745,038	$1,244,215	1.67
3	70,040	$9.50	$665,380	$1,124,490	1.69
Current	75,000	$10.00	$750,000		

4.3 Developing and Using Estimating Factors (cont)

Example of Estimating Factor Development (cont)

Step 5. Select the relationship that best predicts the dependent variable.

It appears from the information above, that there is a relatively consistent relationship between Contract Price and the Estimated Direct Labor Cost, price is between 1.67 and 1.69 times the Estimated Direct Labor Cost. The average ratio is 1.68 --

$$\frac{1.68 + 1.67 + 1.69}{3} = 1.68$$

We can therefore expect reasonably accurate estimates from using a factor of 1.68.

Step 6. Document your findings.

Your documentation of CER development should include the information from the 6-step process above. Exact documentation requirements will vary with the analysis involved.

Using an Estimating Factor in Estimated Development

Once you calculate an estimating factor, you can use it to estimate should-pay price for similar product. Using the 1.68 factor from the guard contract example, you can calculate a should-pay price for the current year. Using this factor, the best estimate of a reasonable price would be $1,260,000, as shown in the table below:

A	B	C	D	E	F
Year	**Estimated Direct Labor Hours**	**SCA Minimum Labor Rate**	**Estimated Direct Labor Cost**	**Contract Price**	**Price to Direct Labor Cost Ratio**
Current	75,000	$10.00	$750,000	$1,260,000	1.68

Given the data above, you should be reasonably confident of your estimate, because the range of ratios is only from 1.67 to 1.69. Even without statistical analysis, that range might be useful in establishing a range of reasonable prices.

High side:	1.69	X	$750,000	=	$1,267,500
Mean:	1.68	X	$750,000	=	$1,260,000
Low side:	1.67	X	$750,000	=	$1,252,500

4.4 Developing and Using Estimating Equations

Section Introduction

Not all estimating relationships lend themselves to the use of simple estimating factors. If there is a substantial element of the cost or price being estimated that is not related to the independent variable (i.e., there is a "fixed cost" that is not associated with the independent variable), you should consider using a linear estimating equation. If the relationship is not linear, consider a nonlinear estimating equation.

Example of Estimating Equation Development

Assume that you are analyzing the costs proposed for the construction of a new house and decide to develop a CER to support your analysis. Development should follow the 6-step CER process described in the chapter Introduction

Step 1 **Define the dependent variable.**

The objective is to estimate the cost of a new base housing model.

Step 2. **Select independent variables to be tested for developing estimates of the dependent variable.**

A variety of house characteristics could be used to estimate cost. These include such characteristics as square feet of living area, exterior wall surface area, number of baths, and others.

Step 3. **Collect data concerning the relationship between the dependent and independent variables.**

Base Housing Model	Unit Cost	Baths	Sq. Ft. Living Area	Sq. Ft. Exterior Wall Surface
Burger	$166,500	2.5	2,800	2,170
Metro	$165,000	2.0	2,700	2,250
Suburban	$168,000	3.0	2,860	2,190
Executive	$160,500	2.0	2,440	1,990
Ambassador	$157,000	2.0	1,600	1,400
New Home		2.5	2,600	2,100

4.4 Developing and Using Estimating Equations (cont)

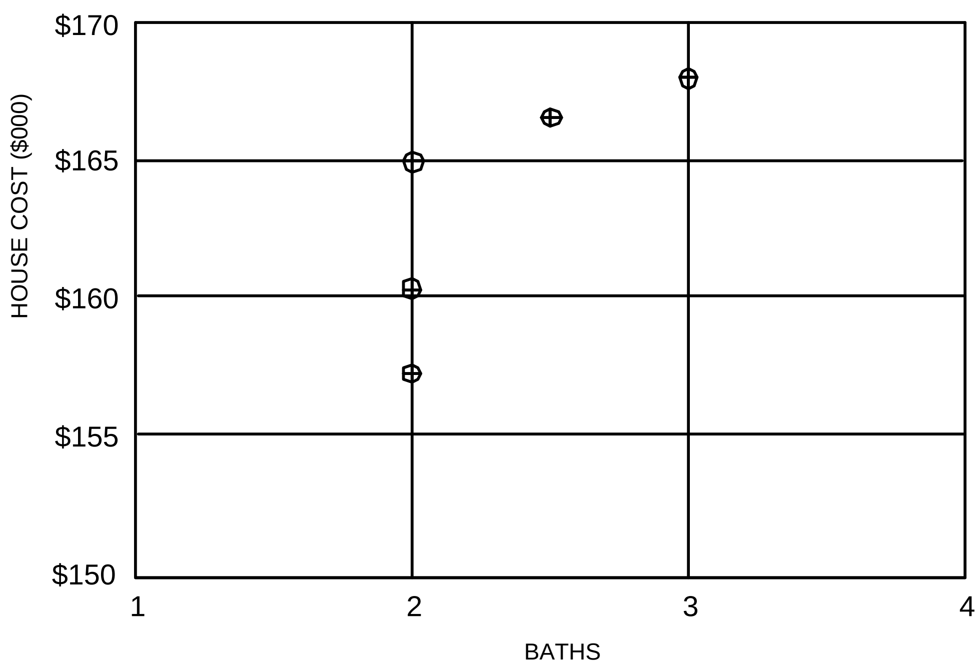
Example of Estimating Equation Development (cont)

Step 4. Explore the relationship between the dependent and independent variables.

Analysis of the relationship between the independent variable and house price could be performed using many different techniques. In this situation most analysts would use regression analysis. However, here we will use graphic analysis to demonstrate the thought process involved. Three independent variables will be tested: number of baths, living area, and exterior wall surface area.

Step 5. Select the relationship that best predicts the dependent variable.

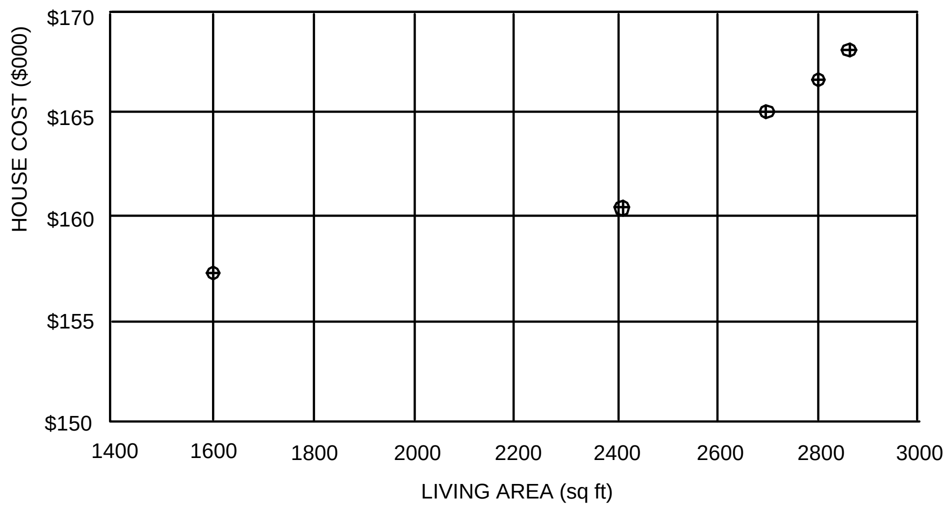
Price and the Number Of Baths. This graph appears to demonstrate that the number of baths is not a good estimating tool, because three houses with a nearly $8,000 price difference have the same number of baths.



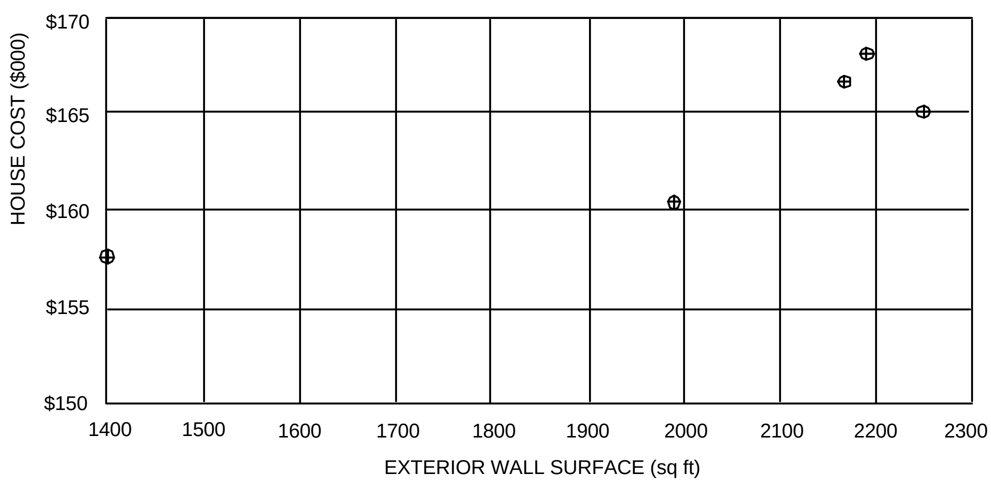
4.4 Developing and Using Estimating Equations (cont)

Example of Estimating Equation Development (cont)

Price and Square Feet Of Living Area. This graph appears to depict a relationship.



Price and Exterior Wall Surface Area. This graph also appears to depict a relationship.



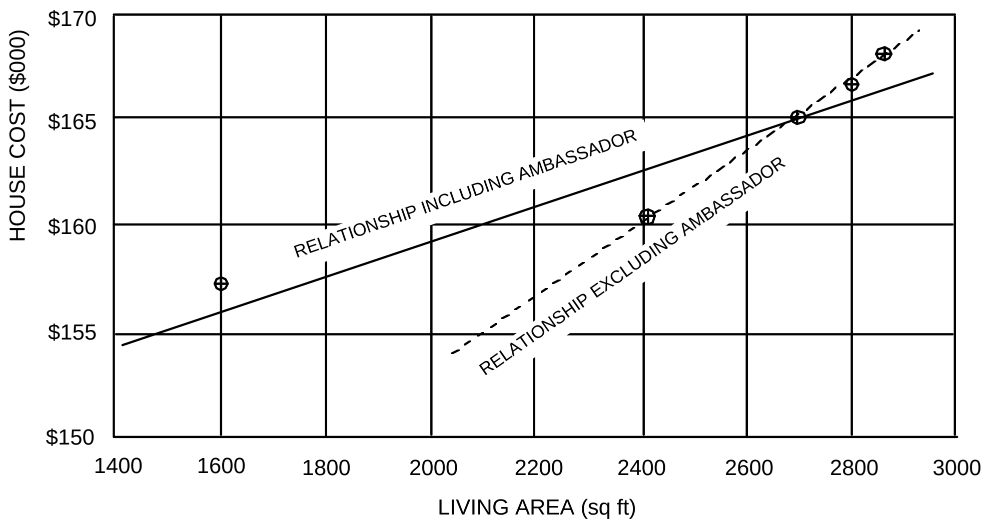
4.4 Developing and Using Estimating Equations (cont)

Example of Estimating Equation Development (cont)

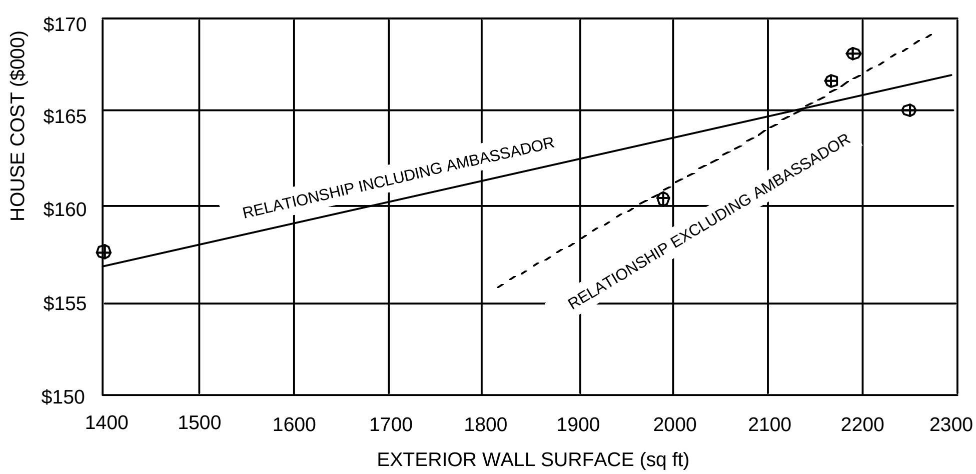
Visually Fitting a Straight Line

Based on the initial analysis, it appears that square feet of living area and exterior wall surface have the most potential for development of a CER. The two graphs below depict efforts to fit a straight line through the observed values. Note that both graphs demonstrate efforts to fit a line with and without using the data from the ambassador model.

Price and Living Area.



Price and Exterior Wall Surface Area.



4.4 Developing and Using Estimating Equations (cont)

Example of Estimating Equation Development (cont)

Consider Analysis Results and Other Data.

Viewing both of these relationships, we might question whether the Ambassador model data should be included in developing our CER. In developing a CER, you need not use all available data if all data is not comparable. However, you should not eliminate data just to get a better-looking relationship. After further analysis, we find that the Ambassador's size is substantially different from the other houses for which we have data and the house for which we are estimating. This substantial difference in size might logically affect relative construction cost. Based on this information, you might decide not to consider the Ambassador data in CER development.

Final Analysis.

If you exclude the Ambassador data, you find that the fit of a straight-line relationship of cost to the exterior wall surface is improved. The relationship between cost and square feet of living area is even closer, almost a straight line. If you had to choose one relationship, you would probably select living area over exterior wall surface because living area has so much less variance from the trend line.

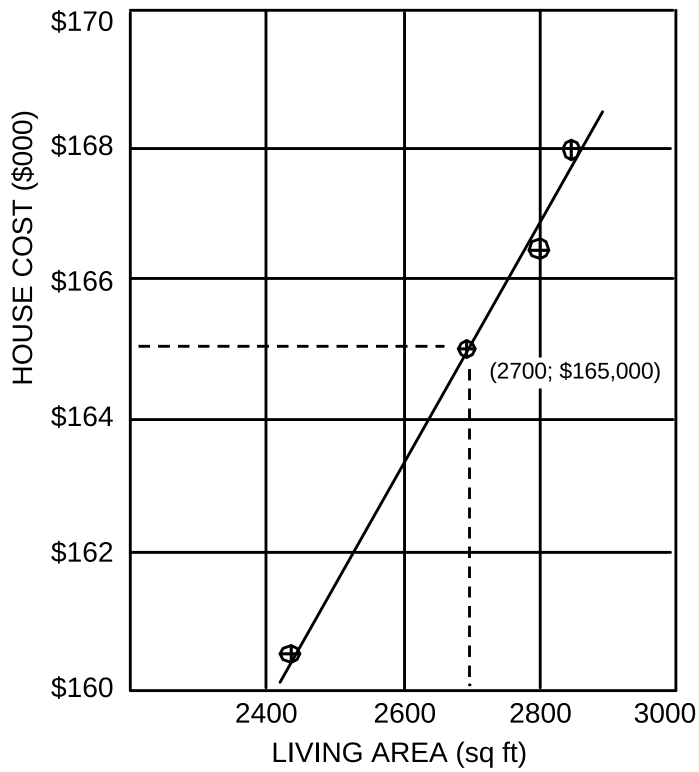
If the analysis of these relationships did not reveal a useful predictive relationship, you might consider combining two or more of the relationships already explored or exploring new relationships. However, since the relationship between living area and price is so close, we may reasonably use it for our CER.

4.4 Developing and Using Estimating Equations (cont)

Example of Estimating Equation Development (cont)

Step 6. Document your findings.

In documenting our findings, we can relate the process involved in selecting living area for price estimation. We may then present the graph, shown on the next page, developed as an estimating tool.



We might also convert the graphic relationship to a mathematical one. This could be done by following the procedures identified in the section on developing a line-of-best-fit.

The cost estimating relationship (CER) would be:

Y = $117,750 + ($17.50 X Sq Ft of Living Area)

4.4 Developing and Using Estimating Equations (cont)

Using an Estimating Equation to Estimate Cost

Once developed, you can use an estimating equation to contract cost or price in similar circumstances.

For example, applying our new CER to the estimation of cost for our new 2,600 square-foot house, we would estimate:

Y = \$117,750 + (\$17.50 X 2,600)
Y = \$117,750 + \$45,500
Y = \$163,250 estimated price

CERs, like most other tools of cost analysis, MUST be used with buyer judgment. Judgment is required to evaluate the historical relationships in the light of new technology, new design, and other similar factors. Therefore, a knowledge of the factors involved in CER development is essential to proper application of the CER. Blind use of any tool can lead to disaster.

4.5 Identify Issues and Concerns

Questions to Consider in Analysis

As you perform price or cost analysis, consider the issues and concerns identified in this section as you consider use of a cost estimating relationship.

- **Does the available information verify the existence and accuracy of the proposed relationship?**

  Technical personnel can be helpful in analyzing the technical validity of the relationship. Audit personnel can be helpful in verifying the accuracy of any contractor data and analysis.

- **Is there a trend in the relationship?**

  For example, the cost of rework is commonly estimated as a factor of production labor. As production continues, the production effort should become more efficient and produce fewer defective units which require repair. The factor should decrease over time. You should also consider the following related questions: Is the rate distorted by one bad run? What is being done to control the rate? What else can be done?

- **Is the CER used consistently?**

  If an offeror uses a CER to propose an element of cost, it should be used in all similar proposals. Since the CER can be used to estimate the average value, some jobs should be expected to incur more cost, others less. With a valid CER you assume the variances will be minor and will average out across all contracts. To use a CER in some cases and a discrete estimate in others destroys it usefulness by over or understating costs across all proposals (e.g. using the average unless a discrete estimate is lower/higher negates the averaging out of cost across all contracts and is clearly unfair to one of the contracting parties)?

- **Has the CER been consistently accurate in the past?**

  No matter how extensive the cost data or how sophisticated the analysis technique, if a CER does not do a good job of accurately projecting cost, then it is not a useful tool.

4.5 Identify Issues and Concerns (cont)

Questions to Consider in Analysis (cont)

- **How current is the CER?**

  Even the most accurate CER needs to be reviewed and updated. While the time interval between updates will differ with CER sensitivity to change, in general a CER should be updated and reviewed at least annually. A CER based on a moving average should be updated whenever new data become available.

- **Would another independent variable be better for developing and applying a CER?**

  If another independent variable would consistently provide a more accurate estimate, then it should be considered. However, remember that the CER may be applicable to other proposals, not just yours. It is possible that a relationship which works well on your contract would not work well across the entire contract population. When assessing CER validity, you should consider all affected contracts.

- **Is the CER a self-fulfilling prophecy?**

  A CER is intended to project future cost. If the CER simply "backs in" to a rate that will spread the cost of the existing capacity across the affected contracts, then the CER is not fulfilling its principle function. If you suspect that a CER is being misused as a method of carrying existing resources, you should consider a should-cost type review on the functions represented by the CER.

- **Would use of a detailed estimate or direct comparison with actuals from a prior effort produce more accurate results?**

  Development of a detailed estimate can be time consuming and costly but the application of the engineering principles required is particularly valuable in estimating cost of efficient and effective contract performance.

CHAPTER 5

Regression Analysis

Learning Objective

At the End of This Chapter	At the end of this chapter you will be able to: Classroom Learning Objective 5/1 Correctly use regression analysis in estimating and analyzing contract cost or price.

5.0 Chapter Introduction

In This Chapter

In this chapter, you will learn to use regression analysis in developing cost estimating relationships and other analyses based on a straight-line relationship even when the data points do not fall on a straight line.

5.0 Chapter Introduction

Line-of-Best-Fit

The straight-line is one of the most commonly used and most valuable tools in both price and cost analysis. It is primarily used to develop cost estimating relationships and to project economic trends. Unfortunately, in contract pricing the data points that are used in analysis do not usually fall exactly on a straight line. Much of the variation in a dependent variable may be explained by a linear relationship with an independent variable, but there are usually random variations that cannot be explained by the line. The goal in establishing a line-of-best-fit is to develop a predictive relationship that minimizes the random variations. This can be done visually with a graph and a ruler, but the visual line-of-best-fit is an inexact technique and has limited value in cost or price analysis. Regression analysis is commonly used to analyze more complex relationships and provide more accurate results.

This chapter will focus on simple regression (two-variable linear regression); in which a single independent variable (X) is used to predict the value of a single dependent variable (Y). The dependent variable will normally be either price or cost (e.g., dollars or labor hours), the independent variable will be a measure related to the product (supply or service) being acquired. It may be a physical characteristic of the product, a performance characteristic of the product, or an element of cost to provide the product.

In some situations, you may need regression analysis tools that are more powerful than simple regression. Multiple regression (multivariate linear regression) and curvilinear regression are variations of simple regression that you may find useful. The general characteristics of both will be addressed later in the chapter.

5.1 Identifying Situations for Use

Cost Estimating Relationship Development and Analysis

Regression analysis is one of the techniques most commonly used to establish cost estimating relationships (CERs) between independent variables and cost or price. If you can use regression analysis to quantify a CER, you can then use that CER to develop and analyze estimates of product cost or price.

Indirect Cost Rate Analysis

FAR 31.203

Indirect costs are costs that are not directly identified with a single final cost objective (e.g., contract item), but identified with two or more final cost objectives or an intermediate cost objective. In addition, minor direct costs may be treated as indirect costs if the treatment is consistently applied to all final cost objectives and the allocation produces substantially the same results as treating the cost as a direct cost.

Because indirect costs are not directly identified with a single final cost objective, they must be accumulated into logical cost pools and allocated to final cost objectives using indirect cost rates (e.g., overhead and general and administrative expense rates). The base used to allocate indirect costs should be selected to permit allocation of the cost pool on the basis of the benefits accruing to the various cost objectives.

Regression analysis is commonly used to quantify the relationship between the indirect cost rate base and pool over time. If you can quantify the relationship, you can then use that relationship to develop and analyze indirect cost rate estimates.

5.1 Identifying Situations for Use (cont)

Time-Series Analysis

You can use regression analysis to analyze trends that appear to be related to time. It is particularly useful when you can identify and adjust for other factors that affect costs or prices (e.g., quantity changes) to isolate the effect of inflation/deflation for analysis. The most common applications of this type are forecasting future wage rates, material costs, and product prices.

In time-series analysis, cost or price data are collected over time for analysis. An estimating equation is developed using time as the independent variable. The time periods are normally weeks, months, quarters, or years. Each time period is assigned a number (e.g., the first month is 1, the fourth month is 4, etc.). All time periods during the analysis must be considered, whether or not data were collected during that period.

Time does not cause costs or prices to change. Changes are caused by a variety of economic factors. Do not use time-series analysis when you can identify and effectively measure the factors that are driving costs or prices. If you can identify and measure one or more key factors, you should be able to develop a better predictive model than by simply analyzing cost or price changes over time. However, if you cannot practically identify or measure such factors, you can often make useful predictions by using regression analysis to analyze cost or price trends over time.

Just remember that regression analysis will not automatically identify changes in a trend (i.e., it cannot predict a period of price deflation when the available data trace a trend of increasing prices). As a result, regression analysis is particularly useful in short-term analysis. The further you predict into the future, the greater the risk.

5.2 Developing and Using a Simple Regression Equation

Simple Regression Model

The simple regression model is based on the equation for a straight line:

$Y_C = A + BX$

Where:

Y_C = the calculated or estimated value for the dependent (response) variable

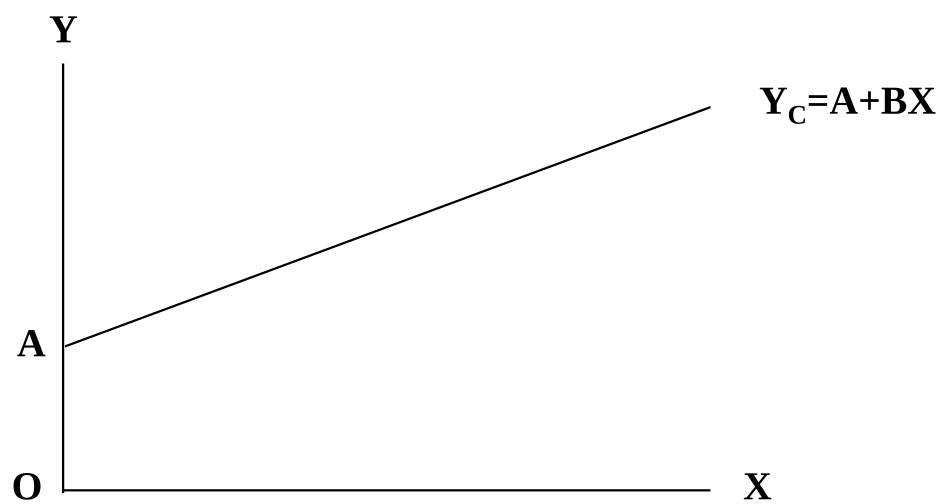
A = the Y intercept, the value of Y when X = 0

X = the independent (explanatory) variable

B = the slope of the line, the change in Y divided by the change in X, the value by which Y changes when X changes by one.

For a given data set, A and B are constants. They do not change as the value of the independent variable changes. Y_C is a function of X. Specifically, the functional relationship between Y_C and X is that Y_C is equal to A plus the product of B times X.

The following figure graphically depicts the regression line:



5.2 Developing and Using a Simple Regression Equation (cont)

Steps for Developing a Simple Regression Equation

To develop a regression equation for a particular set of data, use the following 5-step least-squares-best-fit (LSBF) process:

Step 1. Collect the historical data required for analysis.

Identify the X and Y values for each observation.

X = Independent variable

Y = Dependent variable

Step 2. Put the data in tabular form.

Step 3: Compute $\bar{X}$ and $\bar{Y}$.

$$\bar{X} = \frac{\Sigma X}{n} \qquad \bar{Y} = \frac{\Sigma Y}{n}$$

Step 4. Compute the slope (B) and the Y intercept (A).

$$B = \frac{\Sigma XY - n\,\bar{X}\,\bar{Y}}{\Sigma X^2 - n\bar{X}^2}$$

$$A = \bar{Y} - B\bar{X}$$

Step 5. Formulate the estimating equation.

$$Y_C = A + BX$$

5.2 Developing and Using a Simple Regression Equation (cont)

Simple Regression Equation Development Example

Assume a relationship between a firm's direct labor hours and manufacturing overhead costs based on the use of direct labor hours as the allocation base for manufacturing overhead. Develop an estimating equation using direct labor hours as the independent variable and manufacturing overhead as the dependent variable. You want to estimate the indirect cost pool assuming that 2,100 manufacturing direct labor hours will be needed to meet production requirements for 19X8.

Step 1. Collect the Historical Data Required for Analysis.

HISTORICAL DATA		
Year	**Manufacturing Direct Labor Hours**	**Manufacturing Overhead**
19X2	1,200	$ 73,000
19X3	1,500	$ 97,000
19X4	2,300	$128,000
19X5	2,700	$155,000
19X6	3,300	$175,000
19X7	3,400	$218,000
19X8	2,100 (Est)	

5.2 Developing and Using a Simple Regression Equation (cont)

Simple Regression Equation Development Example (cont)

Step 2: Put The Data In Tabular Form.

X = Manufacturing direct labor hours in hundreds of hours (00s)

Y = Manufacturing overhead in thousands of dollars ($000s)

	TABULAR PRESENTATION				
	X	**Y**	**XY**	X^2	Y^2
	12	73	876	144	5,329
	15	97	1,455	225	9,409
	23	128	2,944	529	16,384
	27	155	4,185	729	24,025
	33	175	5,775	1,089	30,625
	34	218	7,412	1,156	47,524
Column Totals	144	846	22,647	3,872	133,296

Step 3: Compute $\bar{X}$ and $\bar{Y}$.

$$\bar{X} = \frac{\Sigma X}{n} = \frac{144}{6} = 24$$

$$\bar{Y} = \frac{\Sigma Y}{n} = \frac{846}{6} = 141$$

5.2 Developing and Using a Simple Regression Equation (cont)

Simple Regression Equation Development Example (cont)

Step 4. Compute the slope (B) and the intercept (A).

$$B = \frac{\Sigma XY - n\overline{X}\,\overline{Y}}{\Sigma X^2 - n\overline{X}^2}$$

$$= \frac{22{,}647 - 6(24)(141)}{3{,}872 - 6(24)^2}$$

$$= \frac{22{,}647 - 20{,}304}{3{,}872 - 3{,}456}$$

$$= \frac{2{,}343}{416}$$

$$= 5.6322$$

$$A = \overline{Y} - B\overline{X}$$

$$= 141 - 5.6322(24)$$

$$= 141 - 135.1728$$

$$= 5.8272$$

Step 5: Formulate the estimating equation.

Substitute the calculated values for A and B into the equation Yc = A + BX:

Yc = 5.8272 + 5.6322X

Where:

Yc = manufacturing overhead ($000's)
X = manufacturing direct labor hours (00's)

Example of Estimate Using Simple Regression Equation

Estimate manufacturing overhead given an estimate for manufacturing direct labor hours of 2,100:

$$Y_C = 5.8272 + 5.6322X$$
$$= 5.8272 + 5.6322(21)$$
$$= 5.8272 + 118.2762$$
$$= 124.1034 \text{ thousand dollars}$$

Convert Y_C to dollars: $124,103

5.3 Analyzing Variation in the Regression Model

Assumptions of the Regression Model

The assumptions listed below enable us to calculate unbiased estimators of the population regression function coefficients (∂ and ß) and to use these in predicting values of Y given X. You should be aware of the fact that violation of one or more of these assumptions reduces the efficiency of the model, but a detailed discussion of this topic is beyond the purview of this text. Assume that all the assumptions have been met.

1. For each value of X there is an array of possible Y values which is normally distributed about the regression line.

2. The mean of the distribution of possible Y values is on the regression line. That is, the expected value of the error term is zero.

3. The standard deviation of the distribution of possible Y values is constant regardless of the value of X (this is called "homoscedasticity").

4. The error terms are statistically independent of each other. That is, there is no serial correlation.

5. The error term is statistically independent of X.

NOTE: These assumptions are very important, in that they enable us to construct predictions around our point estimate.

Variation in the Regression Model

Recall that the purpose of regression analysis is to predict the value of a dependent variable given the value of the independent variable. The LSBF technique yields the best single line to fit the data, but you also want some method of determining how good this estimating equation is. In order to do this, you must first partition the variation.

Total Variation.

The sum of squares total (SST) is a measure of the total variation of Y. SST is the sum of the squared differences between the observed values of Y (Y_i) and the mean of Y ($\overline{Y}$).

$$SST = \sum (Y_i - \overline{Y})^2$$

5.3 Analyzing Variation in the Regression Model (cont)

Variation in the Regression Model (cont)

While the above formula provides a clear picture of the meaning of SST, you can use the following formula to speed SST calculation:

$$SST = \Sigma Y^2 - \bar{Y}\Sigma Y$$

Total variation can be partitioned into two variations categories: explained and unexplained.

Explained Variation

The sum of squares regression (SSR) is a measure of variation of Y that is explained by the regression equation. SSR is the sum of the squared differences between the calculated value of Y (Y_c) and the mean of Y ($\bar{Y}$).

$$SSR = \sum(Y_c - \bar{Y})^2$$

You can use the following formula to speed SSR calculation:

$$SSR = B(\Sigma XY - \bar{X}\Sigma Y)$$

Unexplained Variation

The sum of squares error (SSE) is a measure of the variation of Y that is not explained by the regression equations. SSE is the sum of the squared differences between the observed values of Y (Y_i) and the calculated value of Y (Y_c). This is the random variation of the observations around the regression line.

$$SSE = \sum(Y_i - Y_c)^2$$

You can use the following formula to speed SSE calculation:

$$SSE = \Sigma Y^2 - A\Sigma Y - B\Sigma XY$$

5.3 Analyzing Variation in the Regression Model (cont)

Analysis of Variance

Variance is equal to variation divided by degrees of freedom (df). In regression analysis, df is a statistical concept that is used to adjust for sample bias in estimating the population mean.

Mean square regression (MSR)

$$MSR = \frac{SSR}{df}$$

For simple regression, the value of df for calculating MSR is always one (1). As a result, in simple regression, you can simplify the equation for MSR to read:

$$MSR = \frac{SSR}{1}$$
$$MSR = SSR$$

Mean square error (MSE)

$$MSE = \frac{SSE}{df}$$

In simple regression, df for calculating MSE is always n - 2. As a result, in simple regression, you can simplify the equation for MSE to read:

$$MSE = \frac{SSE}{n-2}$$

The terms used to analyze variation/variance in the regression model are commonly summarized in an Analysis of Variance (ANOVA) table.

ANOVA TABLE			
Source	Sum of Squares	df	Mean Square**
Regression	SSR	1	MSR
Error	SSE	n-2	MSE
Total	SST	n-1	
**Mean Square = Sum of Squares/df			

5.3 Analyzing Variation in the Regression Model (cont)

Constructing an ANOVA Table for the Manufacturing Overhead Example

Before you can calculate variance and variation, you must use the observations to calculate the statistics in the table below. Since we already calculated these statistics to develop the regression equation to estimate manufacturing overhead, we will begin our calculations with the values in the table below:

STATISTIC	VALUE
$\sum X$	144
$\sum Y$	846
$\sum XY$	22,647
$\sum X^2$	3,872
$\sum Y^2$	133,296
$\bar{X}$	24
$\bar{Y}$	141
A	5.8272
B	5.6322
n	6

Calculate SST.

$$\begin{aligned} SST &= \Sigma Y^2 - \bar{Y}\Sigma Y \\ &= 133{,}296 - 141(846) \\ &= 133{,}296 - 119{,}286 \\ &= 14{,}010 \end{aligned}$$

Calculate SSR.

$$\begin{aligned} SSR &= B(\Sigma XY - \bar{X}\Sigma Y) \\ &= 5.6322[22{,}647 - 24(846)] \\ &= 5.6322[22{,}647 - 20{,}304] \\ &= 5.6322[2{,}343] \\ &= 13{,}196.24 \text{ (round to 13,196 for this example)} \end{aligned}$$

5.3 Analyzing Variation in the Regression Model (cont)

Constructing an ANOVA Table for the Manufacturing Overhead Example (cont)

Calculate SSE.

$$
\begin{aligned}
SSE &= \Sigma Y^2 - A\Sigma Y - B\Sigma XY \\
&= 133,296 - 5.8272(846) - 5.6322(22,647) \\
&= 133,296 - 4929.81 - 127,552.43 \\
&= 813.76 \text{ (round to 814 for this example)}
\end{aligned}
$$

Calculate MSR.

$$
\begin{aligned}
MSR &= SSR \\
&= 13,196
\end{aligned}
$$

Calculate MSE.

$$
\begin{aligned}
MSE &= \frac{SSE}{n-2} \\
&= \frac{814}{6-2} \\
&= \frac{814}{4} \\
&= 203.5 \text{ (round to 204 for this example)}
\end{aligned}
$$

Combine the calculated values into an ANOVA table.

ANOVA TABLE			
Source	Sum of Squares	df	Mean Square**
Regression	13,196	1	13,196
Error	814	4	204
Total	14,010	5	
**Mean Square = Sum of Squares/df			

Check Sum of Squares total.

Assure that value for SST is equal to SSR plus SSE.

$$
\begin{aligned}
SST &= SSR + SSE \\
14,010 &= 13196 + 814 \\
14,010 &= 14,010
\end{aligned}
$$

5.4 Measuring How Well the Regression Equation Fits the Data

Statistics Used to Measure Goodness of Fit

How well does the equation fit the data used in developing the equation? Three statistics are commonly used to determine the "goodness of fit" of the regression equation:

Coefficient of Determination
Standard Error of the Estimate
T-test for Significance of the Regression Equation

Calculating the Coefficient of Determination

Any computer software designed to fit a line using regression analysis will also provide the coefficient of determination for that line. The coefficient of determination (r^2) measures the strength of the association between independent and dependent variables (X and Y).

The range of r^2 is between zero and one.

$$0 \leq r^2 \leq 1$$

An r^2 of zero indicates that there is no relationship between X and Y. An r^2 of one indicates that there is a perfect relationship between X and Y. As r^2 closer it gets to 1, the better the regression line fits the data set.

In fact, r^2 is the ratio of explained variation (SSR) to total variation (SST). An r^2 of .90 indicates that 90 percent of the variation in Y has been explained by its relationship with X; that is, 90 percent of the variation in Y has been explained by the regression line.

$$r^2 = \frac{SSR}{SST}$$

For the manufacturing overhead example:

$$r^2 = \frac{13{,}196}{14{,}010}$$

$$= .94$$

This means that approximately 94% of the variation in manufacturing overhead (Y) can be explained by its relationship with manufacturing direct labor hours (X).

5.4 Measuring How Well the Regression Equation Fits the Data (cont)

Standar Error of the Estimate.

The standard error of the estimate (SEE) is a measure of the accuracy of the estimating (regression) equation. The SEE indicates the variability of the observed (actual) points around the regression line (predicted points). That is, it measures the extent to which the observed values (Y_i) differ from their calculated values (Y_c). Given Assumptions 1 and 2 of the regression model (for each value of X there is an array of possible Y values which is normally distributed about the regression line and the mean of this distribution (Y_c) is on the regression line), the SEE is interpreted in a way similar to the way in which the standard deviation is interpreted. That is, given a value for X, we would generally expect the following intervals (based on the Empirical Rule):

$Y_c \pm 1$ SEE to contain approximately 68 percent of the total observations (Y_i)

$Y_c \pm 2$ SEE to contain approximately 95 percent of the total observations (Y_i)

$Y_c \pm 3$ SEE to contain approximately 99 percent of the total observations (Y_i)

The SEE is equal to the square root of the MSE.

$$SEE = \sqrt{MSE}$$

For the manufacturing overhead example:

$$\begin{aligned} SEE &= \sqrt{204} \\ &= 14.28 \end{aligned}$$

5.4 Measuring How Well the Regression Equation Fits the Data (cont)

Steps for Conducting the T-test for the Significance of the Regression Equation

The regression line is derived from a sample. Because of sampling error, it is possible to get a regression relationship with a rather high r^2 (say > 80 percent) when there is no real relationship between X and Y. That is, when there is no statistical significance. This phenomenon will occur only when you have very small sample data sets. You can test the significance of the regression equation by applying the T-test. Applying the T-test is a 4-step process:

Step 1: Determine the significance level (α).

$$\alpha = 1 - \text{confidence level}$$

The selection of the significance level is a management decision; that is, management decides the level of risk associated with an estimate which it will accept. In the absence of any other guidance, use a significance level of .10.

Step 2: Calculate T.

Use the values of MSR and MSE from the ANOVA table:

$$T = \sqrt{\frac{MSR}{MSE}}$$

Step 3:Determine the table value of t.

From a t Table, select the t value for the appropriate degrees of freedom (df). In simple regression:

$$df = n - 2$$

Step 4: Compare T to the t Table value.

Decision rules:

If T > t, use the regression equation for prediction purposes. It is likely that the relationship is significant.

If T < t, do not use the regression equation for prediction purposes. It is likely that the relationship is not significant.

If T = t, a highly unlikely situation, you are theoretically indifferent and may elect to use or not use the regression equation for prediction purposes.

5.4 Measuring How Well the Regression Equation Fits the Data (cont)

Conducting the T-test for the Significance of the Regression Equation for the Manufacturing Overhead Example

To demonstrate use of the T-test, we will apply the 4-step procedure to the manufacturing overhead example:

Step 1: Determine the significance level (α).

Assume that we have been told to use $\alpha = .05$.

Step 2: Calculate T.

$$T = \sqrt{\frac{MSR}{MSE}}$$

$$= \sqrt{\frac{13{,}196}{204}}$$

$$= \sqrt{64.69}$$

$$= 8.043$$

Step 3: Determine the table value of t.

The partial table below is an excerpt of a t table.

$$\begin{aligned} df &= n - 2 \\ &= 6 - 2 \\ &= 4 \end{aligned}$$

PARTIAL t TABLE	
df	**t**
- - - - - - - - - - -	
2	4.303
3	3.182
4	2.776
5	2.571
6	2.447
- - - - - - - - - - -	

Reading from the table, the appropriate value is 2.776.

5.4 Measuring How Well the Regression Equation Fits the Data (cont)

Conducting the T-test for the Significance of the Regression Equation for the Manufacturing Overhead Example (cont)

Step 4: Compare T to the t Table value.

Since T (8.053) > t (2.776), use the regression equation for prediction purposes. It is likely that the relationship is significant.

NOTE: There is not normally a conflict in the decision indicated by the T-test and the magnitude of r^2. If r^2 is high, T is normally > t. A conflict would occur only in a situation where there are very few data points. In those rare instances where there is a conflict, you should accept the decision indicated by the T-test. It is a better indicator than r^2 because it takes into account the sample size (n) through the degrees of freedom (df).

5.5 Calculating and Using a Prediction Interval

Formulating the Prediction Interval

You can develop a regression equation and use it to calculate a point estimate for Y given any value of X. However, a point estimate alone does not provide enough information for sound negotiations. You need to be able to establish a range of values which you are confident contains the true value of the cost or price which you are trying to predict. In regression analysis, this range is known as the prediction interval.

For a regression equation based on a small sample, you should develop a prediction interval, using the following equation:

$$Y_C \pm tSEE\sqrt{1 + \frac{1}{n} + \frac{(X - \bar{X})^2}{\Sigma X^2 - n\bar{X}^2}}$$

NOTE: The prediction interval will be smallest when $X = \bar{X}$. When $X = \bar{X}$, the final term under the radical sign becomes zero. The greater the difference between X and $\bar{X}$, the larger the final term under the radical sign and the larger the prediction interval.

5.5 Calculating and Using a Prediction Interval (cont)

Constructing a Prediction Interval for the Manufacturing Overhead Example

Assume that we want to construct a 95 percent prediction interval for the manufacturing overhead estimate at 2,100 manufacturing direct labor hours. Earlier in the chapter, we calculated Y_C and the other statistics in the following table:

STATISTIC	VALUE
Y_C	124.1034
t (Use n - 2 df)	2.776
SEE	14.27
$\bar{X}$	24
X^2	3,872

Using the table data, you would calculate the prediction interval as follows:

$$Y_C \pm t\,SEE\sqrt{1+\frac{1}{n}+\frac{(X-\bar{X})^2}{\Sigma X^2 - n\bar{X}^2}}$$

$$124.1034 \pm 2.776(14.27)\sqrt{1+\frac{1}{6}+\frac{(21-24)^2}{3{,}872-6(24)^2}}$$

$$124.1034 \pm 39.6135\sqrt{1+.1667+\frac{(-3)^2}{3{,}872-3{,}456}}$$

$$124.1034 \pm 39.6135\sqrt{1.1667+\frac{9}{416}}$$

$$124.1034 \pm 39.6135\sqrt{1.1667+.0216}$$

$$124.1034 \pm 39.6135\sqrt{1.1883}$$

$$124.1034 \pm 39.6135(1.0901)$$

$$124.1034 \pm 43.1827$$

When X = 21 the prediction interval is: $80.9207 \leq Y \leq 167.2861$.

Prediction Statement: We would be 95 percent confident that the actual manufacturing overhead will be between $80,921 and $167,286 at 2,100 manufacturing direct labor hours.

5.6 Identifying the Need for Advanced Regression Analysis

Section Introduction

In simple regression analysis, you use a single independent variable (X) to estimate the dependent variable (Y), and the relationship is assumed to form a straight line. This is the most common form of regression analysis used in contract pricing. However, when you need more than one independent variable to estimate cost or price, you should consider multiple regression (or multivariate linear regression). When you expect that a trend line will be a curve instead of a straight line, you should consider curvilinear regression.

A detailed presentation on how to use multiple regression or curvilinear regression is beyond the scope of this text. However, you should have a general understanding of when and how these techniques can be applied to contract pricing. When you identify a situation that seems to call for the use of one of these techniques, consult an expert for the actual analysis. You can obtain more details on the actual use of these techniques from advanced forecasting texts.

Multiple Regression Situation

Multiple regression analysis assumes that the change in Y can be better explained by using more than one independent variable. For example, suppose that the Region Audit Manager (RAM) wants to determine the relationship between main-frame computer hours, field-audit hours expended in audit analysis, and the cost reduction recommendations sustained during contract negotiations.

COMPUTER HOURS	FIELD AUDIT HOURS	SUSTAINED REDUCTION
1.4	45	$290,000
1.1	37	$240,000
1.4	44	$270,000
1.1	45	$250,000
1.3	40	$260,000
1.5	46	$280,000
1.5	47	$300,000

It is beyond the purpose of this text to demonstrate how a multivariate equation would be developed using this data. However, we will describe the elements of the multivariate equation and the results of a regression analysis.

5.6 Identifying the Need for Advanced Regression Analysis (cont)

Three-Variable Linear Equation

Multiple regression can involve any number of independent variables. To solve the audit example above, we would use a three-variable linear equation -- two independent variables and one dependent variable.

$$Y_C = A + B_1X_1 + B_2X_2$$

Where:

Y_C = the calculated or estimated value for the dependent (response) variable

A = the Y intercept, the value of Y when $X_1 = 0$ and $X_2 = 0$

X_1 = the first independent (explanatory) variable

B_1 = the slope of the line related to the change in X_1, the value by which Y changes when X_1 changes by one.

X_2 = the second independent (explanatory) variable

B_2 = the slope of the line related to the change in X_2, the value by which Y changes when X_2 changes by one.

Results of Audit Data Three-Variable Linear Regression Analysis

Using the above data on audit analysis and negotiated reductions, an analyst identified the following three variables:

X_1 = Computer Hours

X_2 = Field Audit Hours

Y = Cost Reductions Sustained

The results of analysts analysis are shown in the following table:

REGRESSION RESULTS		
Predictor Variable	**Equation**	r^2
Computer Hours	$Y = A + BX_1$	.82
Field Audit Hours	$Y = A + BX_2$	.60
Comp Hrs and Field Audit Hrs	$Y = A + B_1X_1 + B_2X_2$	.88

5.6 Identifying the Need for Advanced Regression Analysis (cont)

Results of Audit Data Three-Variable Linear Regression Analysis (cont)

You can see from the r^2 values in the above table that Computer Hours explains more of the variation in Cost Reduction Recommendations Sustained than is explained by Field Audit Hours. If you had to select one independent variable, you would select Computer Hours. However, the combination of the two independent variables in multiple regression explains more of the variation in Cost Reduction Recommendations Sustained than the use of Computer Hours alone. The combination produces a stronger estimating tool.

Curvilinear Regression Analysis

In some cases, the relationship between the independent variable(s) may not be linear. Instead, a graph of the relationship on ordinary graph paper would depict a curve. You cannot directly fit a line to a curve using regression analysis. However, if you can identify a quantitative function that transforms a graph of the data to a linear relationship, you can then use regression analysis to calculate a line of best fit for the transformed data.

COMMON TRANSFORMATION FUNCTIONS	EXAMPLES
Reciprocal	$\frac{1}{X}$
Square Root	$\sqrt{X}$
Log-Log	$\log X$
Power	X^2

For example, improvement curve analysis (presented later in this text) uses a special form of curvilinear regression. While it can be used in price analysis and material cost analysis, the primary use of the improvement curve is to estimate labor hours. The curve assumes that less cost is required to produce each unit as the total units produced increases. In other words, the firm becomes more efficient as the total units produced increases.

5.6 Identifying the Need for Advanced Regression Analysis (cont)

Curvilinear Regression Analysis (cont)

There are many improvement curve formulations but one of the most frequently used is:

$$Y = AX^B$$

Where:

Y = Unit cost (in hours or dollars of the Xth unit)

X = Unit number

A = Theoretical cost of the first unit

B = Constant value related to the rate of efficiency improvement

Obviously, this equation does not describe a straight line. However, using the logarithmic values of X and Y (log-log transformation), we can transform this curvilinear relationship into a linear relationship for regression analysis. The result will be an equation in the form:

$$\log Y = \log A + B\log X$$

Where:

logY = the logarithmic value of Y

logA = the logarithmic value of A

logX = the logarithmic value of X

We can then use the linear equation to estimate the logarithmic value of Y, and from that Y.

5.7 Identifying Issues and Concerns

Questions to Consider in Analysis

As you perform price/cost analysis, consider the issues and concerns identified in this section, whenever you use regression analysis.

Does the r^2 value indicate a strong relationship between the independent variable and the dependent variable?

The value of r^2 indicates the percentage of variation in the dependent variable that is explained by the independent variable. Obviously, you would prefer an r^2 of .96 over an r^2 of .10, but there is no magic cutoff for r^2 that indicates that an equation is or is not acceptable for estimating purposes. However, as the r^2 becomes smaller, you should consider your reliance on any prediction accordingly.

Does the T-test for significance indicate that the relationship is statistically significant?

Remember that with a small data set, you can get a relatively high r^2 when there is no statistical significance in the relationship. The T-test provides a baseline to determine the significance of the relationship.

Have you considered the prediction interval as well as the point estimate?

Many estimators believe that the point estimate produced by the regression equation is the only estimate with which they need to be concerned. The point estimate is only the most likely estimate. It is part of a range of reasonable estimates represented by the prediction interval. The prediction interval is particularly useful in examining risk related to the estimate. A wide interval represents more risk than a narrow interval. This can be quite valuable in making decisions such as contract type selection. The prediction interval can also be useful in establishing positions for negotiation: the point estimate could be the objective, the lower limit of the interval could be the minimum position, and the upper limit could be the maximum position.

5.7 Identifying Issues and Concerns (cont)

Questions to Consider in Analysis (cont)

Are you within the relevant range of data?

The size of the prediction interval increases as the distance from $\overline{X}$ increases. You should put the greatest reliance on forecasts made within the relevant range of existing data. For example, 12 is within the relevant range when you know the value of Y for several values of X around 12 (e.g., 10, 11, 14, and 19).

Are time series forecasts reasonable given other available information?

Time series forecasts are all outside the relevant range of known data. The further you estimate into the future, the greater the risk. It is easy to extend a line several years into the future, but remember that conditions change. For example, the low inflation rates of the 1960s did not predict the hyper-inflation of the 1970s. Similarly, inflation rates of the 1970s did not predict inflation rates of the 1980s and 90s.

Is there a run of points in the data?

A run consisting of a long series of points which are all above or all below the regression line may occur when historical data are arranged chronologically or in order of increasing values of the independent variable. The existence of such runs may be a symptom of one or more of the following problems:

Some factor not considered in the regression analysis is influencing the regression equation (consider multivariate regression)
The equation being used in the analysis does not truly represent the underlying relationship between the variables
The data do not satisfy the assumption of independence
The true relationship may be curvilinear instead of linear (consider curvilinear regression)

5.7 Identifying Issues and Concerns (cont)

Questions to Consider in Analysis (cont)

Have you graphed the data to identify possible outliers or trends that cannot be detected through the mathematics of fitting a straight line?

When you use simple regression, you will fit a straight line through the data. However, the value of the relationship identified may be affected by one or more outliers that should not really be considered in your analysis. These can be easily identified through the use of a graph. Remember though, you cannot discard a data point simply because it does not fit on the line. The graph will help you identify an outlier, but you cannot discard it unless there is a valid reason (e.g., different methods were used for that item).

A graph can also permit you to identify situations where a single simple regression equation is not the best predictor. The graph may reveal that there is more than one trend affecting the data (e.g., the first several data points could indicate an upward trend, the latter data points a downward trend). It could also reveal the true relationship is a curve and not a straight line.

Have you analyzed the differences between the actual and predicted values?

Like the graph, this analysis will provide you information useful in identifying outliers (e.g., there may be one very large variance affecting the relationship). However, the outlier may not be as easy to identify as with a graph because the line will be pulled toward the outlier.

Are you comparing apples with apples?

Regression analysis, like a technique based on historical data, assumes that the past is a good predictor of the future. For example, you might establish a strong relationship between production labor hours and quality assurance labor hours. However, if either production methods or quality assurance methods change substantially (e.g., automation) the relationship may no longer be of any value.

5.7 Identifying Issues and Concerns (cont)

Questions to Consider in Analysis (cont)

How current are the data used to develop the estimating equation? The more recent the data, the more valuable the analysis. Many things may have changed since the out-of-date data were collected. If the contractor fails to provide accurate, current, and complete data,as required the firm is likely violating the requirements of the Truth in Negotiations Act as amended. A price based on defective data is defective pricing.

Would another independent variable provide a better estimating tool? Another equation may produce a better estimating tool. As stated above, you would likely prefer an equation with an r^2 of .96 over one with an r^2 of .10.

Does the cost merit a more detailed cost analysis?

If the cost is high and the r^2 is low, it may merit a more detailed analysis. For example, if you had a relatively low r^2 for a production labor effort, it may be worth considering the use of work measurement techniques in your analysis.

CHAPTER 6

Moving Averages

Learning Objective

At the End of This Chapter	At the end of this chapter you will be able to: Classroom Learning Objective 6/1 Correctly use moving averages in estimating and analyzing contract cost or price.

6.0 Chapter Introduction

In This Chapter

In this chapter, you will learn to use moving averages to estimate and analyze estimates of contract cost and price.

6.0 Chapter Introduction

Time-Series Data	Time series data are a time ordered data set of observations of a variable taken at successive intervals. Examples of time-series data in contract pricing include wage rates and index numbers collected over a period of time.
Time-Series Forecasting Techniques	There are several forecasting techniques which you can apply to time-series data, including: some form of mean (e.g. arithmetic, geometric, weighted, etc.), regression analysis, moving averages, or exponential smoothing. You have already learned about using regression analysis for time-series analysis. The next most popular analysis technique is the moving average. This chapter will present the two types of moving averages that are commonly used in Government contracting, the single moving average and the double moving average.
Single Moving Average	If you cannot identify or you cannot measure an independent variable that you can use to estimate a particular dependent variable, your best estimate is often an average (mean) of past observations. The single moving average builds on this principle by defining the number of observations that you will consider. It assumes that the recent past is the best predictor of the future.

6.0 Chapter Introduction

Single Moving Average (cont)

In a single moving average, data collected over two or more time periods (normally at least three) are summed and divided by the number of time periods. That average then becomes a forecast for future time periods. As data from a new time period is added, data from the earlier time periods are dropped from the average calculation. For example, a 12-month moving average uses data from the most recent 12 months. A 6-month moving average uses data from the most recent six months. You must determine the appropriate number of time periods to consider in the analysis. You can use any time period, but monthly data is the most common.

Double Moving Average

If you believe that there is a trend in the data, you can use a double moving average. Trend in the data means that the data tends to either increase or decrease over time. A double moving average requires that you calculate a moving average and then calculate a second moving average using the averages from your first moving average as observations.

6-Step Procedure for Using Moving Averages

When using moving averages, you should use the following 6-step procedure in your analysis:

Step 1. Collect data.

Step 2. Determine which moving average model to use.

- No time-series trend -- use a single moving average.
- Time-series trend -- use a double moving average.

Step 3. Develop 1-period forecasts using different averaging periods to compare with actual observations to evaluate accuracy.

Step 4. Evaluate 1-period forecast accuracy using mean absolute deviations (MADs) between forecasts and actual observations.

Step 5. Select the averaging period found to produce the most accurate results.

Step 6. Use the moving average in forecasting.

6.1 Identify Situations for Use

Situations for Use

You can use moving averages in any situation where you are attempting to forecast a variable and you cannot identify or you cannot measure an independent variable except time that appears to be related to changes in the variable. In contract pricing, moving averages are often used to:

- **Develop estimating rates and factors.**

For example. Most production operations involve substantial amounts of material. When material is used, there is normally some amount of scrap -- material that can no longer be used for its intended purpose. This can include:

◊ Waste from production operations (e.g., sheet metal left over after shapes have been cut from it).
◊ Spoilage (e.g., material that has exceeded its useful shelf life, losses in storage, defective parts, etc.).
◊ Defective parts (e.g., parts that fail inspection during the production process).

Material scrap rates are affected by a variety of factors including production methods, product design, and materials. Because the specific effect of these variables is difficult to identify and measure, scrap rates are commonly estimated using moving averages.

Rates may be calculated in either dollars or units of material and are commonly calculated in one of the following ways:

$$\frac{\textit{Scrap}\text{ Dollars}}{\text{Total Assembly Material Dollars}} \ \textit{or} \ \frac{\textit{Scrap}\text{ Units}}{\text{Total Assembly Material Units}}$$

$$\frac{\textit{Scrap}\text{ Dollars}}{\text{Material Dollars Purchaed}} \ \textit{or} \ \frac{\textit{Scrap}\text{ Units}}{\text{Material Unit Purchased}}$$

In calculating such estimating factors, you should track the cost element being estimated (scrap) and the factor base separately over the averaging period. Then you can calculate the moving average rate by completing the rate calculation. However, if that is not possible, you can calculate the moving average using scrap for each data period. The major disadvantage of the latter method is that periods of high and low production receive the same weight in analysis.

6.1 Identify Situations for Use (cont)

Situations for Use (cont)

- **Estimate contractor sales volume.**

A variety of factors affect sales volume (e.g., company products, the economy, Government spending, and many others). Estimates should include current contracts, known future contracts, and other known requirements. Estimates must also consider sales which are unknown at the present time. One way of considering such sales is with a moving average based on recent sales data.

- **Estimate contract requirements** .

Many contracts obligate the contractor to meet uncertain requirements. A requirements contract for a particular product may require the contractor to meet all Government demand during the contract period. A maintenance contract may require the contractor to respond to unscheduled service calls. In these and similar situations, a moving average can provide estimates of future requirements based on the recent past.

- **Estimate economic change.**

A moving average can be used to estimate future economic change based on recent history. Wage rates and product price changes (index numbers) can be analyzed using moving averages.

6.2 Determining Which Moving Average Model to Use

General Criteria for Model Selection

There are several moving average models that can be used in contract pricing. The two most commonly used are the single moving average and the double moving average. Your decision on which model to use will depend on whether the data indicate a trend (upward or downward) in the values of the dependent variable. If there is:

- No time-series data trend -- use a single moving average.
- Time-series data trend -- use a double moving average.

Methods to Determine If Data Indicate Dependent Variable Trend

There are three common methods you could use to determine whether or not there is trend in a data set: graphic analysis, regression analysis, and Spearman's rank correlation coefficient.

- **Graphic Analysis.**
  Graphic analysis entails plotting the data (either manually or by computer) and determining by visual inspection whether or not there is trend in the data. The problem with this technique is that it is not consistently accurate. It is particularly difficult to make a decision when there may or may not be a slight trend.

- **Regression Analysis.**
  Regression analysis entails the calculation of a least-squares-best-fit (LSBF) estimating equation using time as the independent variable and testing the significance of the slope using the T-test. Though this is an accurate technique, it is rather tedious even when done using a computer.

- **Spearman's Rank Correlation Coefficient** .
  Spearman's Rank Correlation Coefficient, also known as the Rank Spearman (RANSP) test involves calculation of an RS value and comparing that value with a critical value obtained from a table. This is the test that is most commonly used, because it is accurate and relatively easy to calculate even when done manually. However, before using the RANSP test, assure that the following three conditions have been met:

◊ You must have data from at least four observations.
◊ You must not have reason to suspect a cyclical or seasonal effect.
◊ You must not have reason to suspect that there is a change in trend direction (monotonic trend).

6.2 Determining Which Moving Average Model to Use (cont)

7-Step RANSP Test Process

Use the following 7-step procedure to complete the RANSP test:

Step 1: Put the data in table format in the order in which the values occurred and number the time periods.

Assign a value of one (1) to the oldest time period from which data were obtained. Then assign time-period numbers (t) sequentially to each time period from which data were obtained.

Step 2: Create a new variable R_Y.

R_Y is the rank of variable Y. Assign an R_Y of 1 to the smallest value of Y, an R_Y of 2 to the second smallest a 2, and so forth. In the event of a 2-way tie, sum the next two ranks available, divide by 2 and assign that quotient to both numbers. In the event of an n-way tie, sum the next n ranks available and divide by n.

Step 3: Create a new variable D.

D is the difference between the t value for variable Y and the rank of variable Y:

$$D = t - R_Y$$

Step 4: Square each D value and sum the resulting values.

$$\Sigma D^2$$

Step 5: Calculate RS.

Use the following formula:

$$RS = 1 - \frac{6\Sigma D^2}{n(n^2 - 1)}$$

RS	INTERPRETATION
-1	Perfect Negative Trend
0	No Trend
1	Perfect Positive Trend

6.2 Determining Which Moving Average Model to Use (cont)

7-Step RANSP Test Process (cont)

Step 6: Compare the absolute value of RS, with the critical or table value of RS (RS_{crit}).

The absolute value of RS (|RS|) is the calculated value of RS without consideration of sign (i.e., absolute numbers have no sign -- positive or negative). In the absence of other guidance, use a significance level (α) of .10. A partial RS_{crit} table appears below. See Appendix C for a complete RS_{crit} table.

Step 7: Select the appropriate moving average model.

Use the following criteria:

IF	THEN
$\lvert RS \rvert > RS_{crit}$	Assume that there is trend in the data -- use double moving average.
$\lvert RS \rvert = RS_{crit}$	Indifference point -- use either single or double moving average. Normally, you would use a single moving average, because it is easier to use.
$\lvert RS \rvert < RS_{crit}$	Assume that there is no trend in the data -- use a single moving average.

RANSP Test Example 1

Assume that you have collected historical quarterly wage data and you want to determine if there is trend in the data.

Step 1: Put the data in table format in the order in which the values occurred and number the time periods.

QUARTER (t)		WAGE RATE (Y)
1		$12.50
2		$11.80
3		$12.85
4		$13.95
5		$13.30
6		$13.95
7		$15.00
8		$16.20
9		$16.10

6.2 Determining Which Moving Average Model to Use (cont)

RANSP Test Example 1 (cont)

Step 2: Create a new variable R_Y.

R_Y is the rank of variable Y. The smallest value of Y receives a 1, the second smallest a 2, etc. In the event of an n-way tie, sum the next n ranks available and divide by n.

t	Y	R_Y
1	$12.50	2
2	$11.80	1
3	$12.85	3
4	$13.95	5.5
5	$13.30	4
6	$13.95	5.5
7	$15.00	7
8	$16.20	9
9	$16.10	8

Step 3: Create a new variable D.

For each time period, D is the difference between the t value and the R_Y value for that time period:

$$D = t - R_Y$$

t	Y	R_Y	D
1	$12.50	2	-1.00
2	$11.80	1	1.00
3	$12.85	3	0.00
4	$13.95	5.5	-1.50
5	$13.30	4	1.00
6	$13.95	5.5	.50
7	$15.00	7	0.00
8	$16.20	9	-1.00
9	$16.10	8	1.00

6.2 Determining Which Moving Average Model to Use (cont)

RANSP Test
Example 1 (cont)

Step 4: Square each D value and sum the resulting values ($\sum D^2$).

t	Y	R_Y	D	D^2
1	$12.50	2	-1.00	1.00
2	$11.80	1	1.00	1.00
3	$12.85	3	0.00	0.00
4	$13.95	5.5	-1.50	2.25
5	$13.30	4	1.00	1.00
6	$13.95	5.5	.50	.25
7	$15.00	7	0.00	0.00
8	$16.20	9	-1.00	1.00
9	$16.10	8	1.00	1.00
			Total	7.50

Step 5: Calculate RS.

$$
\begin{aligned}
RS &= 1 - \frac{6\Sigma D^2}{n(n^2 - 1)} \\
&= 1 - \frac{6(7.50)}{9(9^2 - 1)} \\
&= 1 - \frac{45}{9(80)} \\
&= 1 - \frac{45}{720} \\
&= 1 - .0625 \\
&= .9375
\end{aligned}
$$

6.2 Determining Which Moving Average Model to Use (cont)

RANSP Test Example 1 (cont)

Step 6: Compare the absolute value of RS, with the critical or table value of RS (RS_{crit}).

Since there is no guidance to the contrary, use a significance level of .10. Since there are nine observations, RS_{crit} at that level of significance is .4667.

Partial Table—Critical Values of RS			
n	Significance Level (a)		
	.10	.05	.01
8	.5000	.6190	.8095
9	.4667	.5833	.7667
10	.4424	.5515	.7333

Step 7. Select the appropriate moving average model.

RS (.9375) > RS_{crit} (.4667). Assume that there is a trend in the data. Use a double moving average.

RANSP Test Example 2

Again, assume that you have collected historical quarterly wage data and you want to determine if there is trend in the data.

Step 1: Put the data in table format in the order in which the values occurred and number the time periods.

QUARTER (t)	WAGE RATE (Y)
1	$12.70
2	$12.60
3	$12.00
4	$13.00
5	$12.10
6	$12.50
7	$12.80
8	$13.00
9	$12.85

6.2 Determining Which Moving Average Model to Use (cont)

RANSP Test Example 2 (cont)

Step 2: Create a new variable R_Y.

R_Y is the rank of variable Y. The smallest value of Y receives a 1, the second smallest a 2, etc. In the event of an n-way tie, sum the next n ranks available and divide by n.

t	Y	R_Y
1	$12.70	5
2	$12.60	4
3	$12.00	1
4	$13.00	8.5
5	$12.10	2
6	$12.50	3
7	$12.80	6
8	$13.00	8.5
9	$12.85	7

Step 3: Create a new variable D.

$$D = t - R_Y$$

t	Y	R_Y	D
1	$12.70	5	-4.00
2	$12.60	4	-2.00
3	$12.00	1	2.00
4	$13.00	8.5	-4.50
5	$12.10	2	3.00
6	$12.50	3	3.00
7	$12.80	6	1.00
8	$13.00	8.5	-0.50
9	$12.85	7	2.00

6.2 Determining Which Moving Average Model to Use

RANSP Test
Example 2 (cont)

Step 4: Square each D value and sum the resulting values (ΣD^2).

t	Y	R_Y	D	D^2
1	$12.70	5	-4.00	16.00
2	$12.60	4	-2.00	4.00
3	$12.00	1	2.00	4.00
4	$13.00	8.5	-4.50	20.25
5	$12.10	2	3.00	9.00
6	$12.50	3	3.00	9.00
7	$12.80	6	1.00	1.00
8	$13.00	8.5	-0.50	0.25
9	$12.85	7	2.00	4.00
Total				67.50

Step 5: Calculate RS.

$$RS = 1 - \frac{6\Sigma D^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(67.5)}{9(9^2 - 1)}$$

$$= 1 - \frac{405}{9(80)}$$

$$= 1 - \frac{405}{720}$$

$$= 1 - .5625$$

$$= .4375$$

6.2 Determining Which Moving Average Model to Use

RANSP Test
Example 2 (cont)

Step 6: Compare the absolute value of RS, with the critical or table value of RS (RS_{crit}).

Since there is no guidance to the contrary, use a significance level of .10. Since there are nine observations, RS_{crit} at that level of significance is .4667.

Partial Table—Critical Values Of RS			
n	**Significance Level (a)**		
	.10	**.05**	**.01**
8	.5000	.6190	.8095
9	.4667	.5833	.7667
10	.4424	.5515	.7333

Step 7. Select the appropriate moving average model .

RS (.4375) < RS_{crit} (.4667). Assume that there is a no trend in the data. Use a single moving average.

6.3 Evaluating and Using Single Moving Averages

Procedures for Selecting a Single Moving Average for Forecasting

The single moving average is designed to smooth random variation in the estimate. The more data periods you use to calculate a single moving average, the greater the smoothing affect. For example, a 12-period moving average will average out most random variation, because each observation is only one-twelfth of the average. However, a 12-period moving average will be slow to react to a true change in the variable that you are attempting to estimate. On the other hand, a three period moving average will react much faster, because one data point is one-third of the calculation instead of one-twelfth.

No averaging period is best for forecasting in all circumstances, you must identify the best averaging period for each situation:

- **Develop 1-period forecasts using different available periods so that you can compare forecasts with actual observations to evaluate accuracy.**

  ◊ Use at least three periods of data in developing a moving average. You can calculate 3-period moving averages beginning in Period 3. You can calculate 4-period moving averages beginning in Period 4. For any value of n, you can calculate an n-period single moving average beginning in Period n.

  ◊ To conduct a meaningful evaluation of forecast accuracy, you must have at least two forecasts and actual data from the same periods for accuracy evaluation. As a result, the largest number of periods (n) that you can use for developing single moving averages is two less than the total number of observations.

- **Evaluate 1-period forecast accuracy using mean absolute deviations (MADs) between forecasts and actual observations.**

- **Select the averaging period found to produce the most accurate results.**

6.3 Evaluating and Using Single Moving Averages (cont)

Calculations Required for Forecast Development

Develop 1-period forecasts using different available periods so that you can compare forecasts with actual observations to evaluate accuracy.

Calculating Single Moving Averages. Calculate single moving averages for available averaging periods using the following equation:

$$\mathbf{M1_{n,t} = \frac{Y_t + Y_{t-1} + \ldots + Y_{t-n+1}}{n}}$$

Where:

$M1_{n,t}$ = A single n - period moving average calculated in period t

Y_i = An observation in period i of the variable being forecast

n = The number of time periods in the moving average

Developing Forecasts Using Moving Averages. Once you calculate a moving average, you can use that average for forecasting.

$FM1_{n,t,t+h} = M1_{n,t}$

Where:

$FM1_{n,t,t+h}$ = A single, n-period, moving average forecast made in period t for period t + h

n = The number of periods in the moving average

t = The period in which the forecast is made

h = The horizon, the number of periods you are forecasting into the future

---

6.3 Evaluating and Using Single Moving Averages

Developing 1-Period Forecasts for Example 2 Data

Calculating Single Moving Averages. In the previous section, we determined that we should use a single moving average to forecast future wage rates from the data below. Here we will use the data to demonstrate the procedures for single moving average forecast development.

QUARTER (t)	WAGE RATE (Y)
1	$12.70
2	$12.60
3	$12.00
4	$13.00
5	$12.10
6	$12.50
7	$12.80
8	$13.00
9	$12.85

Note that we have observations from nine periods. That means that we can calculate 3-period, 4-period, 5-period, 6-period, and 7-period moving averages. With nine observations, we cannot evaluate forecasts based on a single moving average of more than seven (9 - 2) periods.

3-PERIOD SINGLE MOVING AVERAGE			
Quarter (t)	Actual (Y)	$\sum^3 Y$	M1*
1	$12.70		
2	$12.60		
3	$12.00	$37.30	$12.43
4	$13.00	$37.60	$12.53
5	$12.10	$37.10	$12.37
6	$12.50	$37.60	$12.53
7	$12.80	$37.40	$12.47
8	$13.00	$38.30	$12.77
9	$12.85	$38.65	$12.88

* Complete terminology for these moving averages is $M1_{3,t}$. To save space the term has been simplified to M1.

6.3 Evaluating and Using Single Moving Averages

Developing 1-Period Forecasts for Example 2 Data (cont)

Developing Forecasts Using Moving Averages : Once we calculate a single moving average, we can use that average to develop a forecast. To evaluate the accuracy of each moving average, we forecast one period into the future so that we can compare the forecast with the actual Y value. For example, the moving average from Period 3, becomes the Forecast for Period 4.

3-Period Single Moving Average Forecast				
Quarter (t)	**Actual (Y)**	**$\sum^3 Y$**	**M1**	**FM1****
1	$12.70			
2	$12.60			
3	$12.00	$37.30	$12.43	
4	$13.00	$37.60	$12.53	$12.43
5	$12.10	$37.10	$12.37	$12.53
6	$12.50	$37.60	$12.53	$12.37
7	$12.80	$37.40	$12.47	$12.53
8	$13.00	$38.30	$12.77	$12.47
9	$12.85	$38.65	$12.88	$12.77

** Complete terminology for the forecasts in this column is $FM1_{3,t,t+1}$. To save space the term has been simplified to FM1.

We would develop the 4-period, 5-period, 6-period, and 7-period single moving average forecasts using the same procedure.

4-Period Single Moving Average Forecast				
Quarter (t)	**Actual (Y)**	**$\sum^4 Y$**	**M1**	**FM1**
1	$12.70			
2	$12.60			
3	$12.00			
4	$13.00	$50.30	$12.58	
5	$12.10	$49.70	$12.43	$12.58
6	$12.50	$49.60	$12.40	$12.43
7	$12.80	$50.40	$12.60	$12.40
8	$13.00	$50.40	$12.60	$12.60
9	$12.85	$51.15	$12.79	$12.60

6.3 Evaluating and Using Single Moving Averages

Developing 1-Period Forecasts for Example 2 Data (cont)

5-Period Single Moving Average Forecast				
Quarter (t)	**Actual (Y)**	**∑⁵Y**	**M1**	**FM1**
1	$12.70			
2	$12.60			
3	$12.00			
4	$13.00			
5	$12.10	$62.40	$12.48	
6	$12.50	$62.20	$12.44	$12.48
7	$12.80	$62.40	$12.48	$12.44
8	$13.00	$63.40	$12.68	$12.48
9	$12.85	$63.25	$12.65	$12.68

6-Period Single Moving Average Forecast				
Quarter (t)	**Actual (Y)**	**∑⁶Y**	**M1**	**FM1**
1	$12.70			
2	$12.60			
3	$12.00			
4	$13.00			
5	$12.10			
6	$12.50	$74.90	$12.48	
7	$12.80	$75.00	$12.50	$12.48
8	$13.00	$75.40	$12.57	$12.50
9	$12.85	$76.25	$12.71	$12.57

7-Period Single Moving Average Forecast				
Quarter (t)	**Actual (Y)**	**∑⁷Y**	**M1**	**FM1**
1	$12.70			
2	$12.60			
3	$12.00			
4	$13.00			
5	$12.10			
6	$12.50			
7	$12.80	$87.70	$12.53	
8	$13.00	$88.00	$12.57	$12.53
9	$12.85	$88.25	$12.61	$12.57

6.3 Evaluating and Using Single Moving Averages (cont)

Calculations Required for Evaluating Forecast Accuracy

Evaluate 1-period forecast accuracy using mean absolute deviations (MADs) between forecasts and actual observations.

You can use several different statistics to measure the accuracy of a moving average forecast: the range of the error terms (R_F), the standard error of the forecast (s_F), or the mean absolute deviation of the forecast (MAD_F). Of these three options, the statistic which best combines the qualities of ease of computation and utility is the MAD_F. As a result, it is the statistic most commonly used to evaluate moving average accuracy.

The MAD_F tells us on average how much, in absolute terms, actual values deviated from the forecasted value.

$$MAD_F = \frac{\Sigma|D|}{n}$$

Where:

MAD_F = The mean absolute deviation of the forecast.

$\sum$ = Sigma, the symbol for summation.

Y = The actual value which occurred.

F = The value which we forecasted.

$|D|$ = The absolute value of the deviation (i.e., the difference, without regard to sign) between the actual value which occurred and the value forecasted. $|D| = |Y - F|$.

n = The number of deviations (D's) computed.

6.3 Evaluating and Using Single Moving Averages (cont)

Evaluating Forecast Accuracy for Example 2 Data

Evaluate forecast accuracy of single moving averages calculated using the data in Example 2.

3-PERIOD SINGLE MOVING AVERAGE EVALUATION					
t	**Actual (Y)**	**M1**	**FM1**	**D**	**|D|**
1	12.70				
2	12.60				
3	12.00	12.43			
4	13.00	12.53	12.43	0.57	0.57
5	12.10	12.37	12.53	-0.43	0.43
6	12.50	12.53	12.37	0.13	0.13
7	12.80	12.47	12.53	0.27	0.27
8	13.00	12.77	12.47	0.53	0.53
9	12.85	12.88	12.77	0.08	0.08
Total Absolute Deviation					2.01
Mean Absolute Deviation = $\sum\|D\| \div n = 2.01 \div 6 =$					0.34

4-PERIOD SINGLE MOVING AVERAGE EVALUATION					
t	**Actual (Y)**	**M1**	**FM1**	**D**	**|D|**
1	12.70				
2	12.60				
3	12.00				
4	13.00	12.58			
5	12.10	12.43	12.58	-0.48	0.48
6	12.50	12.40	12.43	0.07	0.07
7	12.80	12.60	12.40	0.40	0.40
8	13.00	12.60	12.60	0.40	0.40
9	12.85	12.79	12.60	0.25	0.25
Total Absolute Deviation					1.60
Mean Absolute Deviation = $\sum\|D\| \div n = 1.60 \div 5 =$					0.32

6.3 Evaluating and Using Single Moving Averages

Evaluating Forecast Accuracy for Example 2 Data (cont)

5-PERIOD SINGLE MOVING AVERAGE EVALUATION					
t	**Actual (Y)**	**M1**	**FM1**	**D**	**|D|**
1	12.70				
2	12.60				
3	12.00				
4	13.00				
5	12.10	12.48			
6	12.50	12.44	12.48	0.02	0.02
7	12.80	12.48	12.44	0.36	0.36
8	13.00	12.68	12.48	0.52	0.52
9	12.85	12.65	12.68	0.17	0.17
Total Absolute Deviation					1.07
Mean Absolute Deviation = ∑|D| ÷ n = 1.07 ÷ 4 =					0.27

6-PERIOD SINGLE MOVING AVERAGE EVALUATION					
t	**Actual (Y)**	**M1**	**FM1**	**D**	**|D|**
1	12.70				
2	12.60				
3	12.00				
4	13.00				
5	12.10				
6	12.50	12.48			
7	12.80	12.50	12.48	0.32	0.32
8	13.00	12.57	12.50	0.50	0.50
9	12.85	12.71	12.57	0.28	0.28
Total Absolute Deviation					1.10
Mean Absolute Deviation = ∑|D| ÷ n = 1.10 ÷ 3 =					0.37

6.3 Evaluating and Using Single Moving Averages

Evaluating Forecast Accuracy for Example 2 Data (cont)

7-PERIOD SINGLE MOVING AVERAGE EVALUATION					
t	**Actual (Y)**	**M1**	**FM1**	**D**	**|D|**
1	12.70				
2	12.60				
3	12.00				
4	13.00				
5	12.10				
6	12.50				
7	12.80	12.53			
8	13.00	12.57	12.53	0.47	0.47
9	12.85	12.61	12.57	0.28	0.28
Total Absolute Deviation					0.75
Mean Absolute Deviation = $\sum\|D\| \div n = .75 \div 2 =$					0.38

Selecting an Averaging Period

Select the averaging period found to produce the most accurate results.

SUMMARY OF MAD COMPUTATIONS	
n	**MAD_F**
3	0.34
4	0.32
5	0.27
6	0.37
7	0.38

The lowest MAD_F in this example was attained using a 5-period single moving average. Accordingly, we would select a 5-period single moving average for forecasting.

6.3 Evaluating and Using Single Moving Averages (cont)

Use a Single Moving Average in Forecasting

Use the moving average in forecasting.

We would use the moving average that produced the lowest MAD_F for forecasting. Based on our evaluation of the data in Example 2, we would use the most recent 5-period to forecast for any future period. For example our forecast for Period 13 would be $12.65.

The selection of the most accurate averaging period for forecast development is essential. For example, using different averaging periods and the data in this example, we could have calculated a wide range of forecasts (for period 13, for example).

FORECAST COMPARISON	
n	**$FM1_{n,9,13}$**
3	$12.88
4	$12.79
5	$12.65
6	$12.71
7	$12.61

Of these possibilities, our analysis indicates $12.65 should be the most accurate forecast.

6.4 Evaluating and Using Double Moving Averages

Procedures for Selecting a Double Moving Average for Forecasting

The double moving average is designed to develop a forecast that smoothes random variation and projects any trend exhibited in the data. As with the single moving average, no averaging period is best for forecasting in all circumstances. You must identify the best averaging period for each situation:

- **Develop 1-period forecasts using different available periods so that you can compare forecasts with actual observations to evaluate accuracy.**

  ◊ Normally, we use at least a three period double moving average. Since a double moving average is a moving average of moving averages, you cannot begin to calculate a 3-period double moving average until Period 5. You can calculate 4-period double moving averages beginning in Period 7. For any value of n, you can calculate an n-period double moving average beginning in Period $2n - 1$.

  ◊ To conduct a meaningful evaluation of forecast accuracy, you must have at least two forecasts and actual data for the same period for accuracy evaluation. As a result, you must have $2n + 1$ data points in order to calculate a double moving average forecast and the related MAD.

- **Evaluate 1-period forecast accuracy using mean absolute deviations (MADs) between forecasts and actual observations.**

- **Select the averaging period found to produce the most accurate results.**

6.4 Evaluating and Using Double Moving Averages

Calculations Required for Forecast Development

Develop 1-period forecasts using available averaging periods so that you can compare forecasts with actual observations to evaluate accuracy.

Calculating Double Moving Averages . Calculate double moving averages for available averaging periods using the following equation:

$$M2_{n,t} = \frac{M1_{n,t} + M1_{n,t-1} + \ldots + M1_{n,t-n+1}}{n}$$

Where:

$M2_{n,t}$ = An n-period double moving average calculated in period t

$M1_{n,t}$ = An n-period single moving average calculated in period t

n = Number of periods in the moving average

Note: You must use the same value of n for calculating both M1 and M2.

6.4 Evaluating and Using Double Moving Averages

Calculations Required for Forecast Development (cont)

Developing Forecasts Using Moving Averages: Once you calculate a double moving average, you can use that average to develop a forecast.

$$FM2_{n,t,t+h} = A_{n,t} + B_{n,t}\, h$$

Where:

$FM2_{n,t,t+h}$ = The, n-period, double moving average forecast made in period t for period t + h

$A_{n,t}$ = The intercept for an n-period double moving average forecast, calculated:

$$A_{n,t} = 2M1_{n,t} - M2_{n,t}$$

$B_{n,t}$ = The slope for an n-period double moving average forecast

$$B_{n,t} = \frac{2}{n-1}(M1_{n,t} - M2_{n,t})$$

n = The number of periods in the moving average

t = The period in which the forecast is made

h = The horizon, the number of periods you are forecasting into the future

Note: There is a unique intercept (A) and slope (B) depending on the value of n and the period in which the forecast is made.

6.4 Evaluating and Using Double Moving Averages

Developing 1-Period Forecasts for Example 1 Data

Calculating Double Moving Averages. In the previous section, we determined that we should use a double moving average to develop a forecast from the data below. Here we will use the data to demonstrate the procedures for double moving average forecast development.

QUARTER (t)	WAGE RATE (Y)
1	$12.50
2	$11.80
3	$12.85
4	$13.95
5	$13.30
6	$13.95
7	$15.00
8	$16.20
9	$16.10

Note that we have observations from nine periods. That means that we can only calculate 3-period and 4-period moving averages, because we need 2n + 1 observations to compute the double moving average and subsequent MAD.

For a 3-period double moving average, the values of M2 are a 3-period moving average of the values of M1. Both the moving average of Y values used to calculate M1 and the moving average of M1 values used t calculate M2 must cover the same number of periods (i.e., both moving averages must have the same n).

3-PERIOD DOUBLE MOVING AVERAGE					
t	**Actual (Y)**	**$\sum^3$Y**	**M1**	**$\sum^3$M1**	**M2**
1	12.50				
2	11.80				
3	12.85	37.15	12.38		
4	13.95	38.60	12.87		
5	13.30	40.10	13.37	38.62	12.87
6	13.95	41.20	13.73	39.97	13.32
7	15.00	42.25	14.08	41.18	13.73
8	16.20	45.15	15.05	42.86	14.29
9	16.10	47.30	15.77	44.90	14.97

6.4 Evaluating and Using Double Moving Averages

Developing 1-Period Forecasts for Example 1 Data (cont)

Developing Forecasts Using Moving Averages. Period 5 is the first period that we can develop a 3-period double moving average forecast, because that is the first period that we have the values of M1 and M2 that we need to make the forecast. In Period 5, we can make a forecast for Period 6 as follows:

$$\begin{aligned}
A_{3,5} &= 2M1_{3,5} - M2_{3,5} \\
&= 2(13.37) - 12.87 \\
&= 26.74 - 12.87 \\
&= 13.87
\end{aligned}$$

$$\begin{aligned}
B_{3,5} &= \frac{2}{3-1}(M1_{3,5} - M2_{3,5}) \\
&= \frac{2}{3-1}(13.37 - 12.87) \\
&= \frac{2}{2}(13.37 - 12.87) \\
&= 13.37 - 12.87 \\
&= .50
\end{aligned}$$

$$\begin{aligned}
FM2_{3,5,1} &= A_{3,5} + B_{3,5}(h) \\
&= 13.87 + .50(1) \\
&= 14.37
\end{aligned}$$

6.4 Evaluating and Using Double Moving Averages

Developing 1-Period Forecasts for Example 1 Data (cont)

This forecast developed for Period 6 and the forecasts using later data that were developed for Period 7, Period 8, and Period 9 are show in the table below.

3-Period Double Moving Average Forecast						
t	**Actual**	**M1**	**M2**	**A**	**B**	**FM2**
1	12.50					
2	11.80					
3	12.85	37.15				
4	13.95	38.60				
5	13.30	13.37	12.87	13.87	0.50	
6	13.95	13.73	13.32	14.14	0.41	14.37
7	15.00	14.08	13.73	14.43	0.35	14.55
8	16.20	15.05	14.29	15.81	0.76	14.78
9	16.10	15.77	14.97	16.57	0.80	16.57

Forecasts developed using a 4-period moving average and the same procedures are shown in the following table. Note that only two forecasts can be made for comparison with actual observations.

4-Period Double Moving Average Forecast						
t	**Actual**	**M1**	**M2**	**A**	**B**	**FM2**
1	12.50					
2	11.80					
3	12.85					
4	13.95	12.78				
5	13.30	12.98				
6	13.95	13.51				
7	15.00	14.05	13.33	14.77	0.48	
8	16.20	14.61	13.79	15.43	0.55	15.25
9	16.10	15.31	14.37	16.25	0.63	15.98

6.4 Evaluating and Using Double Moving Averages

Calculations Required for Evaluating Forecast Accuracy

Evaluate 1-period forecast accuracy using mean absolute deviations (MADs) between forecasts and actual observations.

Here we use the same formula for calculating MAD_F that we used in evaluating the accuracy of single moving averages.

$$MAD_F = \frac{\Sigma|D|}{n}$$

Where:
MAD_F = The mean absolute deviation of the forecast.

Σ = Sigma, the symbol for summation.
Y = The actual value which occurred.
F = The value which we forecasted.
|D| = The absolute value of the deviation (i.e., the difference, without regard to sign) between the actual value which occurred and the value forecasted.

$|D| = |Y - F|$.

n = The number of deviations (D's) computed.

3-PERIOD DOUBLE MOVING AVERAGE FORECAST EVALUATION								
t	**Actual**	**M1**	**M2**	**A**	**B**	**FM2**	**D**	**|D|**
1	12.50							
2	11.80							
3	12.85	37.15						
4	13.95	38.60						
5	13.30	13.37	12.87	13.87	0.50			
6	13.95	13.73	13.32	14.14	0.41	14.37	-0.42	0.42
7	15.00	14.08	13.73	14.43	0.35	14.55	0.45	0.45
8	16.20	15.05	14.29	15.81	0.76	14.78	1.42	1.42
9	16.10	15.77	14.97	16.57	0.80	16.57	-0.47	0.47
Total Absolute Deviation								2.76
Mean Absolute Deviation = $\sum\|D\| \div n = 2.76 \div 4 =$								0.69

6.4 Evaluating and Using Double Moving Averages

Calculations Required for Evaluating Forecast Accuracy (cont)

4-PERIOD DOUBLE MOVING AVERAGE FORECAST EVALUATION								
t	**Actual**	**M1**	**M2**	**A**	**B**	**FM2**	**D**	**|D|**
1	12.50							
2	11.80							
3	12.85							
4	13.95	12.78						
5	13.30	12.98						
6	13.95	13.51						
7	15.00	14.05	13.33	14.77	0.48			
8	16.20	14.61	13.79	15.43	0.55	15.25	0.95	0.95
9	16.10	15.31	14.37	16.25	0.63	15.98	0.12	0.12
Total Absolute Deviation								1.07
Mean Absolute Deviation = $\sum\|D\| \div n = 1.07 \div 2 =$								0.54

Select Averaging Period

Select the averaging period found to produce the most accurate results.

SUMMARY OF MAD COMPUTATIONS	
n	**MAD_F**
3	0.69
4	0.54

The lowest MAD_F in this example was attained using a 4-period single moving average. Accordingly, we would select a 4-period single moving average for forecasting.

6.4 Evaluating and Using Double Moving Averages

Use a Double Moving Average in Forecasting

Use the moving average in forecasting.

Based on our evaluation of the data in Example 1, we would use the 4-period double moving average for forecasting. For example our forecast for Period 13 [four periods (h) into the future] would be $18.77, calculated as follows:

$$
\begin{aligned}
A_{4,9} &= 2M1_{4,9} - M2_{4,9} \\
&= 2(15.31) - 14.37 \\
&= 30.62 - 14.37 \\
&= 16.25
\end{aligned}
$$

$$
\begin{aligned}
B_{4,9} &= \frac{2}{4-1}\,(M1_{4,9} - M2_{4,9}) \\
&= \frac{2}{4-1}\,(15.31 - 14.37) \\
&= \frac{2}{3}\,(.94) \\
&= .63
\end{aligned}
$$

$$
\begin{aligned}
FM2_{4,9,13} &= A_{4,9} + B_{4,9}(h) \\
&= 16.25 + .63(4) \\
&= 16.25 + 2.52 \\
&= 18.77
\end{aligned}
$$

The selection of the most accurate averaging period for forecast development is essential. For example, using different averaging periods and the data in this example, we could have calculated two very different forecasts.

FORECAST COMPARISON	
n	**$FM2_{n,9,13}$**
3	$19.77
4	$18.77

6.5 Identifying Issues and Concerns

Questions to Consider in Analysis

As you perform price or cost analysis, consider the issues and concerns identified in this section, whenever you use moving averages.

- **Is a moving average the best choice for estimate development?**
  When using a moving average, you assume that the trend experienced over time is the best guide available to forecast future variable values. If that assumption is not correct, you should use another technique. Detailed estimates that consider all the facts involved are normally more defensible in negotiations than the result of any estimating relationship. If an independent variable (other than time) can be identified and measured, another comparison technique may provide better results than moving average analysis. For example, estimating parts demand based on sales and usage data would probably produce better results than an estimate based on use of a moving average. A moving average can estimate price changes based on recent periods but it cannot predict a turning point that will alter the historical pattern.

- **Is the type of moving average selected appropriate for the situation?**
  When there is a trend in the data, you should use a double moving average. When there is no trend, you should use a single moving average. If you use a single moving average in a situation where a trend exists, your forecast will not consider the trend.

- **Is the averaging period the best choice for the data?**

  You should select the averaging period that provides the best estimates when tested against actual observations. Take special care in your analysis when the moving average covers a large number or periods (e.g., 12 months). Selection of an average that covers a large number of periods is often appropriate because it dampens the effect of random fluctuation. However, an average that considers a large number of data points will also make it more difficult to identify a trend in the data. Occasionally, an estimator will use a large number of periods to mask a trend in the data. When analyzing estimates made using a moving average, you should look at the raw data and consider appropriate alternative estimating procedures.

- **How far into the future should you forecast?**

  The farther into the future that you forecast, the greater the risk. Remember, that you cannot predict a change in an historical trend with a moving average.

CHAPTER 7

Improvement Curves

Learning Objective

At the End of This Chapter

At the end of this chapter you will be able to:

Classroom Learning Objective 7/1
Correctly use improvement curves in estimating and analyzing contract cost or price.

7.0 Chapter Introduction

In This Chapter

In this chapter, you will learn improvement curve concepts and their application to cost and price analysis.

7.0 Chapter Introduction

Basic Improvement Curve Concept

You may have learned about improvement curves using the name learning curve analysis. Today, many experts feel that the term learning curve implies too much emphasis on learning by first-line workers. They point out that the theory is based on improvement by the entire organization not just first-line workers. Alternative names proposed for the theory include: improvement curve, cost-quantity curve, experience curve, and others. None have been universally accepted. In this text, we will use the term improvement curve to emphasize the need for efforts by the entire organization to make improvements to reduce costs.

Just as there are many names for the improvement curve, there are many different formulations. However, in each case the general concept is that the resources (labor and/or material) required to produce each additional unit declines as the total number of units produced over the items entire production history increases. The concept further holds that decline in unit cost can be predicted mathematically. As a result, improvement curves can be used to estimate contract price, direct labor-hours, direct material cost, or any other recurring contract cost.

7.0 Chapter Introduction

Improvement Curve History

The improvement curve is based on the concept that, as a task is performed repetitively, the time required to perform the task will decrease. Management planners have followed that element of the concept for centuries, but T. P. Wright pioneered the idea that improvement could be estimated mathematically. In February 1936, Wright published his theory in the *Journal of Aeronautical Sciences* as part of an article entitled “Factors Affecting the Cost of Airplanes.” Wright's findings showed that, as the number of aircraft produced in sequence increased, the direct labor input per airplane decreased in a regular pattern that could be estimated mathematically.

During the mobilization for World War II, both aircraft companies and the Government became interested in the theory. Among other considerations, the theory implied that a fixed amount of labor and equipment could be expected to produce larger and larger quantities of defense products as production continued.

After World War II, the Government engaged the Stanford Research Institute (SRI) to study the validity of the improvement curve concept. The study analyzed essentially all World War II airframe direct labor input data to determine whether there was sufficient evidence to establish a standard estimating model. The SRI study validated a mathematical model based on the World War II findings that could be used as a tool for price analysis. However, that model was slightly different than the one originally offered by Wright.

Since World War II, the improvement curve concept has been used by Government and industry to aid in pricing contracts. Over the years, the improvement curve has been used as a contract estimating and analysis tool in a variety of industries including: airframes, electronics systems, machine tools, shipbuilding, missile systems, and depot level maintenance of equipment. Improvement curves have also been applied to service and construction contracts where tasks are performed repetitively.

7.0 Chapter Introduction

Identifying Basic Improvement Curve Theories

Since 1936, many different formulations have been proposed to explain and estimate the improvement that takes place in repetitive production efforts. Of these, the two most popular are the unit improvement curve and the cumulative average improvement curve

Unit Improvement Curve. The unit improvement curve is the model validated by the post-World War II SRI study. The formulation is also known by two other names: Crawford curve, after one of the leaders of the SRI research; and Boeing curve, after one of the firms that first embraced its use.

Unit curve theory can be stated as follows:

> **As the total volume of units produced doubles the cost per unit decreases by some constant percentage.**

The constant percentage by which the costs of doubled quantities decrease is called the rate of learning. The term “slope” in the improvement curve analysis is the difference between 100 percent and the rate of improvement. If the rate of improvement is 20 percent, the improvement curve slope is 80 percent (100 percent - 20 percent). The calculation of slope is described in detail later in the chapter.

Unit curve theory is expressed in the following equation:

$$Y = AX^{B}$$

Where:

Y = Unit cost (hours or dollars) of the X^{th} unit

X = Unit number

A = Theoretical cost (hours or dollars) of the first unit

B = Constant that is related to the slope and the rate of change of the improvement curve. It is calculated from the relationship:

$$B = \frac{\text{Logarithm of the Slope}}{\text{Logarithm of 2}}$$

In calculating B, the slope MUST be expressed in decimal form rather than percentage form. Then B will be a negative #, leading to the decreasing property stated above.

7.0 Chapter Introduction

Identifying Basic Improvement Curve Theories (cont)

Cumulative Average Improvement Curve. The cumulative average improvement curve is the model first introduced by Wright in 1936. Like the unit improvement curve, the cumulative average curve is also known by two other names: Wright Curve, after T.P. Wright; and Northrop Curve, after one of the firms that first embraced its use.

Cumulative average theory can be stated as follows:

> **As the total volume of units produced doubles the average cost per unit decreases by some constant percentage.**

As with the unit improvement curve, the constant percentage by which the costs of doubled quantities decrease is called the rate of improvement. The slope of the improvement curve analysis is the difference between 100 percent and the rate of learning. However, the rate of improvement and the slope are measured using cumulative averages rather than the unit values used in unit improvement curve analysis.

Cumulative average curve theory is expressed in the following equation:

$$\overline{Y} = AX^B$$

Where:

$\overline{Y}$ = Cumulative average unit cost (hours or dollars) of units through the X^{th} unit

All other symbols have the same meaning used in describing the unit improvement curve.

Curve Differences. Note that the only difference between definitions of the unit improvement curve and the cumulative average improvement curve theories is the word **average**. In the unit curve, unit cost is reduced by the same constant percentage. In the cumulative average curve, the cumulative average cost is reduced by the some constant percentage.

The most significant practical difference between the two different formulations is found in the first few units of production. Over the first few units, an operation following the cumulative average curve will experience a much greater reduction in cost (hours or dollars) than an operation following a unit curve with the same slope. In later production, the reduction in cost for an operation following a cumulative average curve will be about the same as an operation following a unit curve with the same slope.

7.0 Chapter Introduction

Identifying Basic Improvement Curve Theories (cont)

Because of the difference in early production, many feel that the unit curve should be used in situations where the firm is fully prepared for production; and the cumulative average curve should be used in situations where the firm is not completely ready for production. For example, the cumulative average curve should be used in situations where significant tooling or design problems may NOT be completely resolved. In such situations, the production of the first units will be particularly inefficient but improvement should be rapid as problems are resolved.

In practice, firms typically use one formulation regardless of differences in the production situation. Most firms in the airframe industry use the cumulative average curve. Most firms in other industries use the unit curve.

7.1 Situations for Use

Situations for Use

The improvement curve cannot be used as an estimating tool in every situation. Situations that provide an opportunity for improvement or reduction in production hours are the types of situations that lend themselves to improvement curve application. Use of the improvement curve should be considered in situations where there is:

- **A high proportion of manual labor.**

  It is more difficult to reduce the labor input when there is limited labor effort, the labor effort is machine paced, or individual line workers only touch the product for a few seconds.

- **Uninterrupted production.**

  As more and more units are produced the firm becomes more adept at production and the labor hour requirements are reduced. If supervisors, workers, tooling, or other elements of production are lost during a break in production, some improvement will also likely be lost.

- **Production of complex items.**

  The more complex the item the more opportunity there is to improve.

- **No major technological change.**

  The theory is based on continuing minor changes in production and in the item itself. However, if there are major changes in technology, the benefit of previous improvement may be lost.

- **Continuous pressure to improve.**

  The improvement curve does not just happen; it requires management effort. The management of the firm must exert continuous pressure to improve. This requires investment in the people and equipment needed to obtain improvement.

7.1 Situations for Use

Factors that Support Improvement

As you examine situations that appear to have potential for improvement curve application, consider management emphasis on the following factors affecting the rate of improvement:

- **Job Familiarization By Workers.**

  As noted earlier, many feel that this element has been overemphasized over the years. Still, workers do improve from repetition and that improvement is an important part of the improvement curve.

- **Improved Production Procedures.**

  As production continues, both workers and production engineers must constantly be on the lookout for better production procedures.

- **Improved Tooling and Tool Coordination.**

  Part of the examination of production procedures must consider the tooling used for production. Tooling improvements offer substantial possibilities for reduction of labor requirements.

- **Improved Work Flow Organization.**

  Improving the flow of the work can substantially reduce the labor effort that does not add value to the product. Needless movement of work in progress can add significant amounts of labor effort.

- **Improved Product Producibility.**

  Management and workers must constantly consider product changes that will make the product easier to produce without degrading the quality of the final product.

- **Improved Engineering Support.**

  The faster production problems can be identified and solved, the less production labor effort will be lost waiting for problem resolution.

- **Improved Parts Support.**
  As production continues, better scheduling should be possible to eliminate or significantly reduce worker time lost waiting for supplies. In addition, production materials more appropriate for production can be identified and introduced to the production process.

7.2 Analyzing Improvement Using Unit Data and the Unit Theory

Section Introduction

In this text, we will only consider application of the unit improvement curve in making initial contract estimates. There are many texts that address other improvement curve theories (e.g., cumulative average improvement curves), as well as many advanced issues such as the effects of contract changes, breaks in production, and retained learning.

Improvement Illustration

To illustrate the effect of the unit curve, assume that the first unit required 100,000 labor-hours to produce. If the slope of the curve is 80 percent slope, the following table demonstrates the labor-hours required to produce units at successively doubled quantities.

Units Produced	Labor-Hours Per Unit at Doubled Quantities	Difference in Labor-Hours Per Unit at Doubled Quantities	Rate of Improvement (%)	Slope of Curve (%)
1	100,000			
2	80,000	20,000	20	80
4	64,000	16,000	20	80
8	51,200	12,800	20	80
16	40,960	10,240	20	80
32	32,768	8,192	20	80

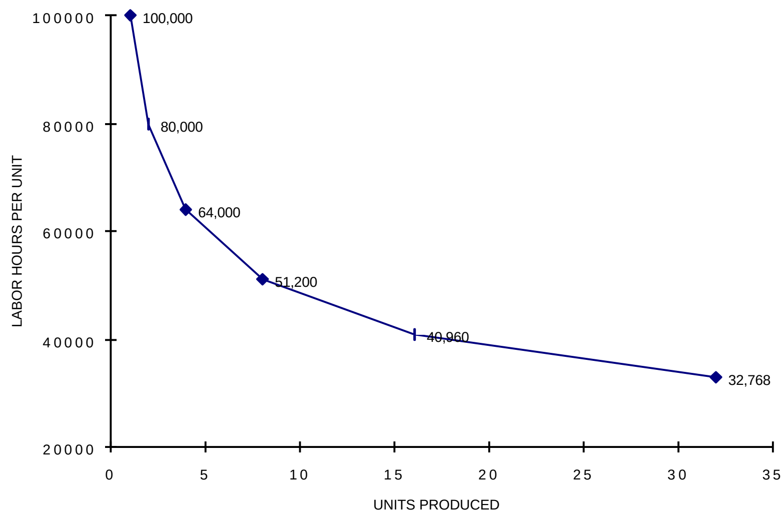
Obviously, the amount of labor-hour reduction between doubled quantities is not constant. The number of hours of reduction between doubled quantities is constantly declining. However, the rate of change or decline remains constant (20 percent).

Also note that the number of units required to double the quantity produced is constantly increasing. Between Unit #1 and Unit #2, it takes only one unit to double the quantity produced. Between Unit #16 and Unit #32, 16 units are needed.

7.2 Analyzing Improvement Using Unit Data and the Unit Theory

Graphing the Data

Rectangular Coordinate Graph. A labor-hour graph of this data curve drawn on ordinary graph paper (rectangular coordinates) becomes a curve as shown in the graph below. In this graph equal spaces represent equal amounts of change. When thinking of numbers in terms of their absolute values, this graph presents an accurate picture, but it is difficult to make an accurate prediction from this curve.



The graph is a curve because the number of hours of reduction between doubled quantities is constantly declining and an increasing number of units are required to double the quantity produced. Note that most of the improvement takes place during the early units of production. The curve will eventually become almost flat. The number of production hours could become quite small but it will never reach zero.

Log-Log Graph. To examine the data and make predictions using unit improvement curve theory, we need to transform the data to logarithms. One way of making the transformation is through the use of log-log graph paper also known as full-logarithmic graph paper.

7.2 Analyzing Improvement Using Unit Data and the Unit Theory (cont)

Log-Log Paper

There are several special elements that we must consider when using log-log graph paper.

- There is already a scale indicated on both the horizontal and vertical axes. Note that there are no zeros. Values can approach zero but never reach it.

- The scale only goes from "1" to "1". Each time the number scale goes from "1" to "1", the paper depicts a cycle. Each "1" moving up on the vertical axis or to the right on the horizontal axis is 10 times the "1" before it. You should mark the actual scale you are using in the margin of the log-log paper before starting to plot points.

  ◊ In improvement curve analysis, always graph the number of the unit produced on the horizontal axis. Assign the first "1" on the left of the page a value of 1 representing the first unit produced. The second "1" is 10. The third "1" is 100. The fourth "1" is 1,000.

  ◊ Always graph the cost in hours or dollars on the vertical axis. The scale will change depending on the data being graphed. The first "1" can be .001, .01, 1, 100, 1,000 or any other integral power of 10. Whatever the value assigned to the first "1," the next "1" is 10 times more, and the next one 10 times more than that. To determine the scale to be used:
    – Estimate the largest number to be plotted or read on the Y axis. This figure will probably be the theoretical cost of the first unit. For example, suppose this is 60,000 hours.
    – Determine the next integral power of ten above this number (e.g., the next integral power above 60,000 is 100,000).
    – Assign this value to the horizontal line at the top of the upper cycle on the Y axis. The horizontal line at the top of the next lower cycle must then represent 10,000 of the same units, and the line at the bottom of the lower cycle represents 1,000.

- On log-log graph paper, the distances between numbers on each axis are equal for equal percentage changes. For example, the distance between "1" and "2" is the same as between "4" and "8;" both represent a 100 percent increase.

7.2 Analyzing Improvement Using Unit Data and the Unit Theory (cont)

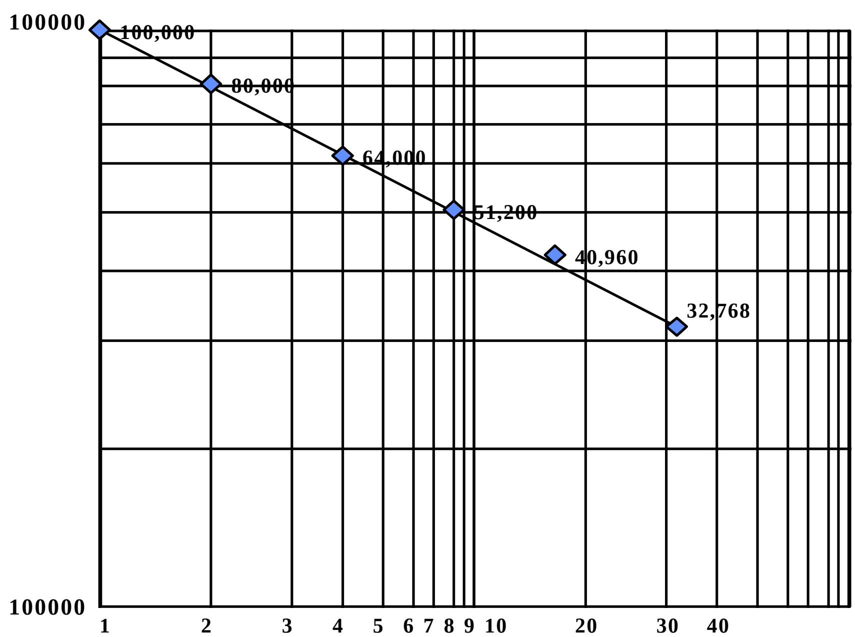
Log-Log Graph

You can obtain surprisingly accurate results from a log-log graph, but your accuracy greatly depends on your graphing technique.

- Always use a sharp pencil.
- Make points plotted on the paper as small as possible and the lines as narrow as possible.
- When the smallest possible point has been marked on the paper, it may easily be lost sight of or confused with a blemish in the paper. To avoid this, draw a small ring around the point. Circles, triangles, and squares are also used to identify points which belong to different sets of data.
- Exercise great care in drawing a line. If it is supposed to go through a point, it should pass exactly through it, not merely close to it.

A graph of the data described in the example above forms a perfectly straight line when plotted on log-log paper. That is, a straight line passes exactly through each of the points. A straight line on log-log paper indicates that the rate of change is constant.

Since improvement curve theory assumes continuing improvement at a constant rate, the straight line becomes an excellent estimating tool. Assuming that improvement will continue at the same rate, the line can be extended to estimate the cost of future units.



7.2 Analyzing Improvement Using Unit Data and the Unit Theory (cont)

Calculating the Theoretical Value of Unit #1

When we discuss improvement curves, we normally describe them in terms of the theoretical value for Unit #1 and the slope of the curve. With these two values, you can use graph paper, tables, or computer programs to estimate the cost of future units.

The value of Unit #1 is referred to as a **theoretical value** (T_1), because in most cases you will not know the actual cost of Unit #1. Instead, T_1 is the value indicated by the line-of-best-fit. On a graph, it is the point where the line-of-best-fit and the vertical line representing Unit #1 intersect. **(Remember, the graph of the improvement curve always begins with Unit #1.)**

Estimating the Slope

The term "slope" as used for improvement curves is a mathematical misnomer. It cannot be related to the definition of slope in a straight line on rectangular coordinates. Instead, the slope of an improvement curve is equal to 100 minus that constant percentage decrease (100 - rate of improvement).

You can calculate the slope of a curve, by dividing the unit cost (Y_X) at some unit (X) into the unit cost (Y_{2X}) at twice the quantity (2X) and multiplying the resulting ratio by 100.

$$\text{Slope} = 100 \left(\frac{Y_{2X}}{Y_X}\right)$$

Therefore, you can measure the slope of an improvement curve drawn on log-log paper by reading a cost (Y_X) at any quantity, X; reading a cost (Y_{2X}) at any quantity, 2X; dividing the second value by the first; and multiplying by 100.

For example, if the number of hours to make Unit #5 is 70 and the number of hours to make Unit #10 is 50, the slope of the improvement curve is:

$$\text{Slope} = 100 \left(\frac{Y_{10}}{Y_5}\right)$$

$$\text{Slope} = 100 \left(\frac{50}{70}\right)$$

Slope = 71.4 percent

7.2 Analyzing Improvement Using Unit Data and the Unit Theory (cont)

Estimating the Slope (cont)

Slope Research Data. The post-war SRI study revealed that many different slopes were experienced by different firms, sometimes by different firms manufacturing the same products. In fact, manufacturing data collected from the World War II aircraft manufacturing industry had slopes ranging from 69.7 percent to almost 100 percent. These slopes averaged 80 percent.

Research by DCAA in 1970 found curves ranging from less than 75 percent to more than 95 percent. The average slope was 85 percent.

Slope Selection and Verification. Unfortunately, information on industry average curves is frequently misapplied by practitioners who use them as a standard or norm. Because each situation is different, you should select a slope based on your analysis of the situation and not on general averages. The order of preference in slope selection is:

- A curve developed from data pertaining to the production of the same product (as we did above).
- The median percentage from a group of curves for items having some similarity to the end item.
- The median percentage from the product category in which the item would most likely be included.

Estimating the Cost/Price of Future Units

The primary purpose for estimating an improvement curve is to predict the cost of future production. The prediction is based on the assumption (not always true) that the past is a good predictor of the future. In terms of the unit improvement curve theory, this assumption means that the unit cost (hours or dollars) of doubled quantities will continue to decrease by the same constant percentage.

Using a graph, you can predict future costs by drawing a line-of-best-fit through the historical data graphed on log-log paper and extending it through the unit for which you wish to make a cost estimate. Estimate cost using the Y value (cost) at the point were the two lines intersect.

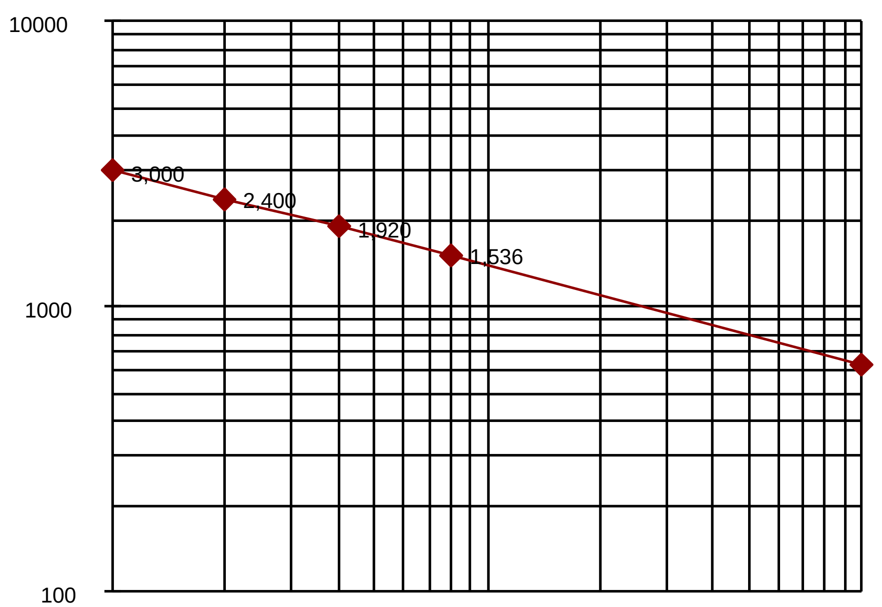
7.2 Analyzing Improvement Using Unit Data and the Unit Theory (cont)

Estimating the Cost/Price of Future Units (cont)

For example, suppose we had the following unit cost data:

UNIT NUMBER	HOURS
1	3,000
2	2,400
4	1,920
8	1,536

Plotting the data on log-log paper, you will observe a straight line with an 80 percent slope.



If you extend the line-of-best-fit to Unit #100, you can estimate the cost of Unit #100. As you can see from the graph, the extended line reveals an estimated cost of approximately 680 hours for Unit #100.

7.3 Analyzing Improvement Using Lot Data and Unit Theory

Accounting System Data

Use of the improvement curve is dependent on available cost data. The relevant accounting or statistical record system must be designed to make relevant data available for analysis. Costs, such as labor-hours per unit or dollars per unit, must be identified with the unit of product.

> **NOTE: It is preferable to use labor-hours rather than dollars since the dollars contain an additional variable, the effect of inflation or deflation, which the labor-hours do not contain** .

Typically accounting systems do not record the cost of individual units. If the firm uses a job-order cost accounting, costs are accumulated on the job order in which the number of units completed are specified and costs are cut-off at the completion of the units. Process cost accounting also yields costs identified with end-item units. In this case, however, the costs are usually assigned to equivalent units produced over a period of time rather than actual units.

Average Unit Cost

To use unit improvement curve theory, you must be able to estimate the cost of a particular unit. Given lot or period costs, the only unit cost that we know is the average cost for the lot or period. However, we have a method for using average costs in improvement curve analysis.

For example, given the following data, we must be able to estimate the cost of an additional 40 units.

LOT NUMBER	LOT SIZE (UNITS)	LOT TOTAL LABOR-HOURS (COST)	LOT AVERAGE LABOR HOURS (COST)
1	6	40,800	6,800
2	9	40,500	4,500
3	15	52,500	3,500

7.3 Analyzing Improvement Using Lot Data and Unit Theory (cont)

Calculating a Lot Plot Point for Graphic Analysis

To graph the lot average unit cost, we must select a corresponding unit number. If we assume that costs go down during the lot, the average cost should occur at the middle of the lot – the lot mid-point. One problem is that the True Lot Mid-Point (the unit where the average cost is incurred) depends on the slope of the improvement curve. Unfortunately, the slope of the curve also depends on the placement of the Lot Mid-Point. The iterative process required to calculate the True Lot Mid-Point for each lot is too cumbersome for manual computation. As a result, we use the following rules of thumb for graphic analysis:

- **For All Lots After The First Lot,** calculate the lot mid-point by **dividing the number of units in the lot by two**. Then add the resulting number to all the units produced prior to the lot to determine where the unit falls in the continuing improvement curve.

  For example, what would be the plot point for a lot made up of units 91 through 100. There are 10 units in the lot, so the middle of the lot would be 5 (10 ÷ 2 = 5). Adding 5 to the 90 units produced prior to the lot, we find that the plot point would be 95.

- **For a First Lot of Less Than 10,** follow the same procedure that you follow for all lots after the first lot. Of course, the lot plot point will equal the lot mid-point because no units will have been produced prior to the first lot.
- **For a First Lot of 10 or More,** calculate the lot mid-point by **dividing the number of units in the lot by three**. This adjustment is necessary to compensate for the rapid decline in cost that takes place in the first lot of production.

7.3 Analyzing Improvement Using Lot Data and Unit Theory (cont)

Calculating a Lot Plot Point for Graphic Analysis (cont)

Given the data above, use a table similar to the following, to calculate the necessary lot plot points and lot average hours:

LOT NO.	LOT SIZE	CUMULATIVE UNITS	LOT MID-POINT	LOT PLOT POINT
1	6	6	3.0	3.0
2	9	15	4.5	10.5
3	15	30	7.5	22.5

You can then use this information to estimate the cost of lots that have not yet been produced. For example, suppose you wanted to estimate the cost of a Lot #4 of 40 units to be produced after the 40 units described above. The final row of the table would be:

4	40	70	20	50

For this example, the lot plot point for Lot #4 would be at Unit #50. You would estimate the average unit cost for the lot using the cost of Unit #50.

Combining Lot Plot Point and Average Unit Cost Calculation

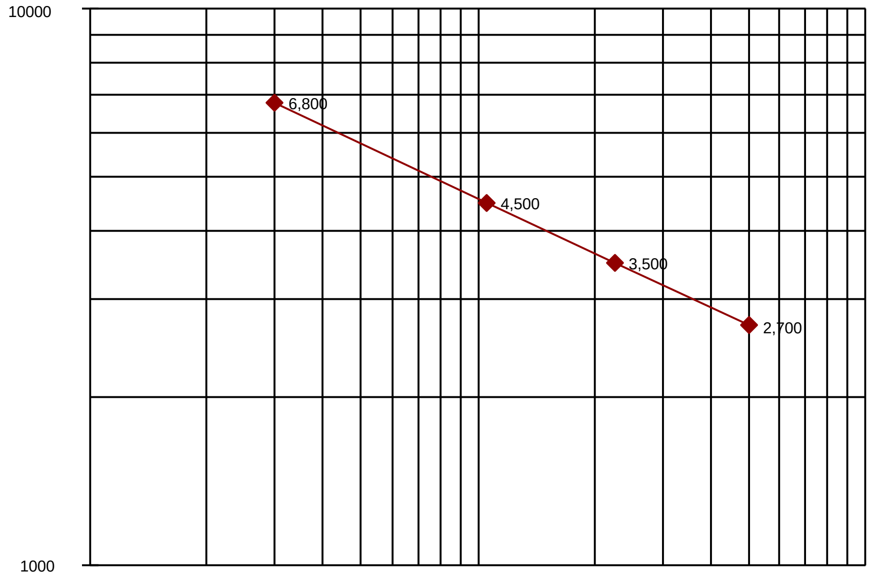
You can combine the calculation for the lot average unit cost and the lot plot point into a single table, as shown below:

LOT NO.	LOT SIZE	CUMULATIVE UNITS	LOT MID-POINT	LOT PLOT POINT	LOT AVERAGE HOURS	LOT TOTAL HOURS
1	6	6	3.0	3.0	6,800	40,800
2	9	15	4.5	10.5	4,500	40,500
3	15	30	7.5	22.5	3,500	52,500
4	40	70	20.0	50.0		

7.3 Analyzing Improvement Using Lot Data and Unit Theory (cont)

Plotting Data on a Log-Log Graph

Plot the average lot cost data (Y) at the corresponding lot plot point (X) on log-log paper and for an improvement curve. Extend the improvement curve through Unit #50, the lot plot point for Lot #4.



On the Y axis, the lot average cost at Unit #50 is approximately 2,700 labor hours. With this information, you can estimate the cost of Lot #4 at 108,000 labor hours (i.e., 2,700 labor hours x 40 units).

7.4 Fitting a Unit Curve

General Points to Consider

Throughout this chapter, we have assumed that all data fit a perfectly straight line. Unfortunately, most data do not fall exactly on a straight line. You need to be able to identify a trend and fit data to that trend. You can visually fit a line using graphic analysis, but most lines-of-best-fit are developed using regression analysis.

Whatever method of analysis you use to fit an improvement curve, if a data point is a significant distance away from the trend set by other data points, look into the cause of the deviation. If your analysis indicates that the data point is not comparable with the rest of the data for some reason, consider adjusting or eliminating the data point from your analysis. However, never eliminate a data point from your analysis simply because it does not fit the apparent trend set by the remaining data.

Graphic Analysis

When visually fitting a straight line, graph the data then draw the line to minimize the distance between the straight line and the data points. Normally, you should give more weight to the larger lots as you fit the straight line.

When fitting a straight line on ordinary graph paper, you know that the line-of-best-fit must go through the average of the X values ($\overline{X}$) and the average of the Y values ($\overline{Y}$). When fitting a line-of-best-fit through improvement curve data on log-log paper, you have no similar fixed reference point. **Without this fixed reference point, even skilled analysts can arrive at very different lines.**

Regression Analysis

Normally, you can obtain more accurate results using regression analysis and a log-log transformation. Using the logarithmic values of X and Y instead of the actual values, the equation of the unit improvement curve ($Y = AX^B$) becomes:

$$\text{Log } Y = \text{Log } A + B(\text{Log } X)$$

The new equation describes a straight line ($Y = A + BX$) relationship. After this transformation, you can use regression analysis to fit a straight line to the data.

7.4 Fitting a Unit Curve (cont)

Regression Analysis (cont)

Improvement curve regression analysis programs differ in several ways including:

- **Use of True Lot Mid-Point.**

  In addition to the accuracy gained from using regression analysis, most improvement curve programs use the true lot mid-point rather than the rule-of-thumb calculations described earlier in this section for graphic analysis. The greatest effect of using the true lot mid-point is in the first lot. Examples of the differences between the rule-of-thumb and true lot mid-points are depicted in the following table:

SELECTED FIRST LOT MID-POINTS				
Units in First Lot	Rule-of-Thumb	True-Lot Mid-Points		
		70% Curve	80% Curve	90% Curve
2	1.00	1.37	1.39	1.4
10	3.33	3.95	4.17	4.36
100	33.33	28.65	32.36	35.43
1,000	333.33	258.15	304.43	340.67
10,000	3333.33	2,495.48	3,002.85	3,384.18

  Differences in calculating the lot mid-point will affect the results of the improvement curve analysis by the placement of the data points for analysis.

- **Method of Regression.**

  Not all improvement curve analysis programs use the same mathematical model for regression analysis. For example, some analysis programs assign a weight to each lot based on the lot size, while others do not. Software using unweighted regression considers all lots (large and small) equally. When weights are assigned to each lot based on lot size, larger lots receive more analysis consideration than smaller lots.

7.4 Fitting a Unit Curve (cont)

Regression Analysis (cont)

- **Measures of Fit.**

  Regardless of the regression model used to develop the line-of-best-fit, virtually all regression analysis software will provide measures of the line's goodness of fit.

  ◊ The primary goodness of fit measure is the coefficient of determination (r^2) for the equation. As described in the chapter on "Using Regression Analysis," the coefficient of determination indicates the portion of variation in Y is explained by the regression line (e.g., an r^2 of .94 indicates that 94 percent of the variation in Y is explained by the relationship between X and Y).

  ◊ Many improvement curve analysis programs also provide the T-test for significance of the regression equation.

- **Graphic Analysis Capability.**

  Many regression analysis programs provide a capability to graph the data and the regression line. For most analysts, this display is one of the strongest tools for identifying anomalies in the data that affect the value of the regression analysis as an estimating tool.

7.5 Estimating Using Unit Improvement Curve Tables

Estimating Choices

Once the cost of Unit #1, in hours or dollars, and the slope of the improvement curve have been established, we can develop estimates of future costs in several ways. You could graph the data on log-log paper and read your estimates from the graph. You could substitute the values into the improvement curve equation. Many analysts use a third choice, improvement curve tables.

Improvement Curve Tables

Improvement curve tables are an expansion of the X^B portion of the basic unit improvement curve equation, $Y = A\ X^B$. The result is recorded as a decimal fraction, which is typically calculated to six or eight decimal places. There is a different table value for each unit and slope. (See the partial table on the next page.)

Unit Estimate. To estimate the price or cost for a specific unit, you can simply multiply the cost of Unit #1 by the appropriate unit factor for the desired unit and slope.

For example, if Unit #1 is 2,000 labor hours, what would be your estimate for Unit #20 if production is expected to follow an 80 percent improvement curve? The table value for Unit #20 and an 80 percent slope is .381208. The estimate would be 762.4 labor hours, calculated as follows:

.381208 x 2,000 labor hours = 762.4 hours

Lot Estimate. To estimate the price of a particular lot, you can use the cum total factors. For example, assume that 10 units have already been produced in Lot #1 and you are attempting to estimate the cost of the 10 units in Lot #2. After Lot #1, 10 units will have been produced. After Lot #2, 20 units will have been produced. To find the cost of Lot #2 (Units #11 to #20), calculate the estimated cost for the first 20 units and subtract from that the estimated cost for the first 10 units. The difference is the cost of Lot #2 (Units #11 to #20).

For example, if Unit #1 is 4,000 labor hours and the improvement curve slope is 80 percent, what would be your estimate for Units #15 to #25? Your estimate should be 15,192.24 labor hours, calculated as follows:

Total Estimate for First 25 Units: 12.308597 x 4,000 = 49,234.39

Less Total Estimate for First 14 Units: 8.092339 x 4,000 = 32,369.36

Estimate for Units #16 to #25:
= 16,865 hrs

7.5 Estimating Using Unit Improvement Curve Tables (cont)

Improvement Curve Tables You can find selected improvement curve tables in the DCAA Contract Audit Manual, Appendix F.

Partial Improvement Curve Table						
	79 Percent		80 Percent		81 Percent	
Unit	Cum Total	Unit	Cum Total	Unit	Cum Total	Unit
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	1.790000	0.790000	1.800000	0.800000	1.810000	0.810000
3	2.478245	0.688245	2.502104	0.702104	2.526065	0.716065
4	3.102345	0.624100	3.142104	0.640000	3.182165	0.656100
5	3.680837	0.578492	3.737741	0.595637	3.795233	0.613068
6	4.224550	0.543713	4.299424	0.561683	4.375245	0.580012
7	4.740494	0.515944	4.833914	0.534490	4.928703	0.553458
8	5.233533	0.493039	5.345914	0.512000	5.460144	0.531441
9	5.707214	0.473681	5.838864	0.492950	5.972892	0.512748
10	6.164223	0.457009	6.315374	0.476510	6.469477	0.496585
11	6.606656	0.442433	6.777485	0.462111	6.951880	0.482403
12	7.036189	0.429533	7.226831	0.449346	7.421690	0.469810
13	7.454188	0.417999	7.664747	0.437916	7.880206	0.458516
14	7.861784	0.407596	8.092339	0.427592	8.328507	0.448301
15	8.259928	0.398144	8.510538	0.418199	8.767503	0.438996
16	8.649429	0.389501	8.920138	0.409600	9.197970	0.430467
17	9.030982	0.381553	9.321821	0.401683	9.620576	0.422606
18	9.405190	0.374208	9.716181	0.394360	10.035902	0.415326
19	9.772580	0.367390	10.103736	0.387555	10.444457	0.408555
20	10.133617	0.361037	10.484944	0.381208	10.846691	0.402234
21	10.488713	0.355096	10.860211	0.375267	11.243003	0.396312
22	10.838235	0.349522	11.229900	0.369689	11.633750	0.390747
23	11.182513	0.344278	11.594336	0.364436	12.019252	0.385502
24	11.521844	0.339331	11.953813	0.359477	12.399798	0.380546
25	11.856497	0.334653	12.308597	0.354784	12.775651	0.375853
26	12.186716	0.330219	12.658929	0.350332	13.147049	0.371398
27	12.512724	0.326008	13.005031	0.346102	13.514210	0.367161
28	12.834725	0.322001	13.347104	0.342073	13.877334	0.363124
29	13.152906	0.318181	13.685335	0.338231	14.236605	0.359271
30	13.467440	0.314534	14.019894	0.334559	14.592192	0.355587

7.6 Identifying Issues and Concerns

Questions to Consider in Analysis

As you perform price or cost analysis, consider the issues and concerns identified in this section, whenever you use an improvement curve.

- **Is improvement curve analysis used when the contract effort involves:**
  - ◊ A significant amount of manual labor in the contract?
  - ◊ Uninterrupted production?
  - ◊ Production of complex items?
  - ◊ No major technological change?
  - ◊ Or should involve, continuous pressure to improve?

- **Is improvement curve use adequately documented?**

  Documentation should include:
  - ◊ A statement describing the improvement curve theory used in developing the estimate.
  - ◊ A summary of related cost data for the product being purchased and any similar products.
  - ◊ A description of how available data were used in estimating the theoretical cost of Unit #1 and the slope of the curve.
  - ◊ A statement on how the improvement curve estimate was used in price or cost analysis.

- **Does the documentation provide a valid base for estimate development?**

  Like CERs, improvement curves are a form of comparison estimate. Unless you are satisfied that the historical data provide a valid base for the use of an improvement curve, estimates based on the curve should be suspect.

7.6 Issues and Concerns (cont)

Questions to Consider in Analysis (cont)

- **Was improvement curve theory properly applied to the available data?**

  Verify the application of the improvement curve to the data available. Remember that different improvement curve models will produce different results.

  For instance, you may find that a unit curve will provide more reasonable results than a cumulative average curve provided by an offeror. Examine the results of both curves when an offeror proposes using a cumulative average curve, because cumulative average curves often conceal significant fluctuations in per unit labor hours.

- **Did any improvement curve analysis isolate costs associated with contract changes and production interruptions?**

  Changes and production interruptions will both have a disruptive effect on improvement. If their effects are not identified and considered in analysis, improvement curve estimates will typically underestimate actual requirements. Random fluctuations around an improvement curve line-of-best-fit should be expected. However, if costs increase or decrease dramatically, you should suspect that the actual costs have been affected by a contract change or a break in production. When you suspect that actual costs are affected by a contract change or break in production, contact the cognizant auditor and Government technical personnel for assistance in your analysis.

  On the other hand, an offeror might overstate the impact of an interruption in production—contending that the interruption has been so long that it will have to start from scratch. However, improvements in unit costs result in part from such factors as better product design, tooling, work methods, and work layout. If these were properly documented, some of the improvement should carry over to the new effort—regardless of the length of the interruption or turnover of personnel.

7.6 Issues and Concerns (cont)

Questions to Consider in Analysis (cont)

- **Does the improvement curve analysis project continued improvement?**

  Occasionally, an offeror will propose "negative learning." In other words, as more units are produced, the cost per unit increases. Do not accept the negative learning argument. If something has significantly changed, consider starting a new curve with a new first unit value and slope.

- **Does the improvement curve estimate include the costs of rework and repair?**

  The effort for rework and repair may or may not be included in the costs projected with the improvement curve. Therefore, you need to determine if these costs are included in the projected costs before allowing any add-on factors for rework or repair.

CHAPTER 8

Work Measurement

Learning Objective

At the End of This Chapter

At the end of this chapter you will be able to:

Classroom Learning Objective 8/1
Correctly use work measurement in estimating and analyzing contract costs.

8.0 Introduction

In This Chapter

In this chapter, you will learn work measurement concepts and their application to cost analysis.

8.0 Introduction (cont)

Work Measurement

Work Measurement involves the use of labor time standards to measure and control the time required to perform a particular task or group of tasks. Most often labor standards are developed and applied in manufacturing operations, however labor standards can be used in estimating and managing the cost of a vast variety of activities including engineering drafting, clerical administration, and janitorial services.

Work Measurement System

DFARS 215.872-1

A Work Measurement System is a management system designed to:

- Analyze the touch labor content of a manufacturing operation.
- Establish labor standards for that operation.
- Measure and analyze variances from those standards.
- Continuously improve both the manufacturing operation and the labor standards used in that operation.

8.0 Introduction (cont)

Work Measurement System Plan

A Work Measurement System Plan is the firm's program for implementing, operating, and maintaining Work Measurement in its operations. The key to an effective System is a Work Measurement Plan that defines the System structure and assigns clear System responsibilities. As a minimum, the Plan should provide guidance on:

- Establishing and maintaining standard accuracy.
- Conducting engineering analyses to improve operations.
- Revising standards and related system data.
- Using labor standards as an input to budgeting, estimating, production planning, and performance evaluation.

Labor Standard Types

A labor standard is a measure of the time it should take for a qualified worker to perform a particular operation. Labor standards are commonly grouped into two types:

- **Engineered Standards** are developed using recognized principles of industrial engineering and work measurement. The standards developed define the time necessary for a qualified worker, working at a pace ordinarily used, under capable supervision, and experiencing normal fatigue and delays, to do a defined amount of work of specified quality when following the prescribed method. As a result, you can use engineered standards to examine contractor operations to estimate the number of labor hours that should be required to efficiently and effectively produce a particular product and to identify any projected contractor variances from that estimate.
- **Non-engineered Standards** are developed using the best information available Engineered Standards without performing the detailed analysis required to develop. Historical costs are commonly used standards that are often a measure of the hours that have been required to complete a task rather than the hours that should be required.

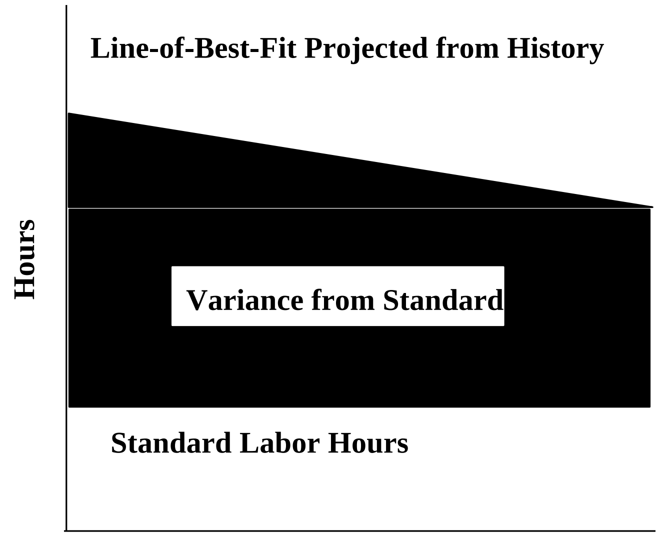
Estimate of Efficient Operation Cost

Labor standards provide information on what it should cost to complete an operation or series of operations in product production. Instead of applying pressure to improve in all areas managers can use this information to identify areas requiring particular management emphasis. The Acquisition Team can use that same information to identify inefficient operations for close scrutiny during contract negotiations.

8.0 Introduction (cont)

Estimate of Efficient Operation Cost (cont)

The log-log graph below presents a line-of-best-fit developed using actual labor-hour history. Note that this line follows the form of the improvement curve. Without labor standards, the firm and the Government would likely project the improvement curve to estimate the labor hours required to produce future units.



However, labor standards provide additional information that can be used in estimate development and analysis. The vertical distance between the labor-hour history and the labor standard represents the variance from the standard. Some of that variance may be "acceptable" because of production inefficiencies that cannot be resolved. However, most of this variance is "questionable." Key elements of the variance include:

- Technical (e.g., manufacturing coordination, engineering design changes, fit problems, design errors, operation sheet errors, tooling errors, work sequence errors, and engineering liaison problems).
- Logistics (e.g., incorrect hardware and parts shortages).
- Miscellaneous (e.g., unusual working conditions, excessive overtime, and excessive fatigue).
- Worker learning (e.g., familiarity with processes and methods).

8.0 Introduction (cont)

Estimate of Efficient Operation Cost (cont)

Variance analysis should identify, categorize, and develop plans to control all variances from standard. Plans will typically concentrate on the operations with the largest variances from standard, because these operations present the greatest opportunity for cost reduction.

Updating Standards

Standards cannot be set and forgotten. Process improvement is one of the central elements of an effective Work Measurement System. As methods improve, the associated labor standards must be updated.

Standards changes will effect the estimating value of all the data based on those standards. For example, some variance analyses may remain valid while other analyses will be rendered meaningless as a result of the change. The System must assure that valid analyses are retained for continued utilization. At the same time, the System must assure that meaningless data are not misused.

8.1 Identifying Situations for Use

General Situations

Consider the use of labor standards whenever contractor employees will be performing the same tasks repetitively over an extended period of time. Labor standard development requires extensive detailed effort. The time and cost required for standards development are prohibitive unless the task will be performed repetitively. On the other hand, when an operation will be performed repetitively, the cost visibility provided by labor standards permits detailed cost evaluation and control, that can result in significant savings to the Government. To be of real value, labor standards must be considered in making key management decisions (e.g., budgeting, estimating, production planning, and performance evaluation).

DoD Policy

For DoD contracts, follow the criteria outlined in the table below:

DFARS 215.872-4

IF YOU	THEN YOU
Are preparing a solicitation or contract for a major weapons system or subsystem with an estimated price in excess of: • $100 million total cost; or • $20 million annually	Should include provisions for Work Measurement Systems.
Are preparing a solicitation or contract for full-scale development exceeding $100 million and you believe that Work Measurement is appropriate (e.g., to assist in transitioning from full-scale development to production)	May include Work Measurement System requirements.
Are acquiring commercial products	Should not include Work Measurement System requirements.
Expect low-volume, nonrepetitive production runs	
Are not requiring submission and certification of cost or pricing data	
Decide that there will be no cost benefit from the imposition of Work Measurement Systems	

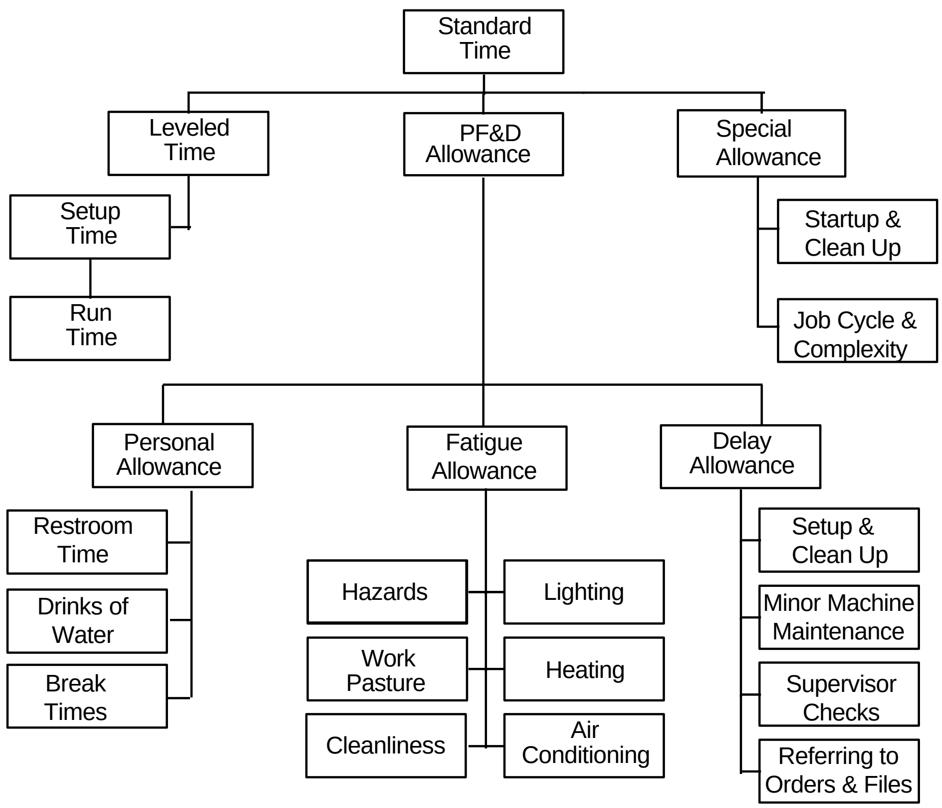
When inserting Work Measurement System provisions in a solicitation or contract, you should insure that they are tailored to the program requirements and compatible with existing contractor technical and management processes and procedures.

8.2 Identifying Elements of a Labor Standard

Key Elements

As a contracting officer, its likely that you will never be required to develop a labor standard. However, you may be called upon to negotiate a contract price based, in part on labor standards. You should know the elements of a standard and how they are developed.

An engineered standard is composed of three elements: leveled time; a personal, fatigue, and delay (PF&D) allowance; and any applicable special allowances. The figure below depicts some of the factors that should be considered in each element.



8.2 Identifying Elements of a Labor Standard (cont)

Leveled Time

Leveled time is the time that a worker of average skill, making an average effort, under average conditions, would take to complete the required task. There are a variety of techniques used in leveled time development, but the four used most commonly are:

- **Time Study.**

  In performing a Time Study, industrial engineers (or other labor analysts) time the effort required to perform a defined task. While it may sound simple, this is a complex process that requires special training and experience. To perform a Time Study, the analyst must:

  ◊ Clearly define and document the work design, including the best design of the work place, tools, tasks, and subtasks.

  ◊ Select a person to be timed. The person selected should be receptive to being timed, experienced in the work methods being used, and familiar with the tasks and subtasks of the work design.

  ◊ Observe and record the time that the selected worker requires to perform each of the subtasks in the work design. Several observations are required to average out random variations and assure that all elements of the work have been considered. The number of observations required will increase as the confidence level desired by the analyst increases and as the variability between observed times increases.

  ◊ Assign a pace rating based on an evaluation of how the ability and effort of the worker being timed compares with those of an average worker. Using the pace ratings, the analyst converts observed times into a leveled time for the subtask.

  ◊ Total subtask times to develop a leveled time for the entire task.

- **Predetermined Leveled Times.**

  Instead of using Time Study to develop a leveled time, the analyst can use predetermined leveled times (also called Predetermined Standards or Basic Motion Standard Data). Predetermined leveled times are established for basic body motions, such as reach, move, turn, grasp, position, release, disengage, and apply pressure. The analyst may obtain them from published standards in tabular or electronic forms, or the firm may develop its own.

8.2 Identifying Elements of a Labor Standard (cont)

Leveled Time (cont)

To use predetermined leveled times, the analyst must:

◊ Clearly define and document the work design, including the best design of the work place, tools, tasks, and subtasks.

◊ Select and document the source of the predetermined leveled times.

◊ Identify and document the basic body motions involved in performing each subtask. Motions for each hand must be specifically identified. The need for precise measurement of complex body motions for each job element may make this method of leveled time development inappropriate for complex tasks with long performance cycle times.

◊ Assign times to the body motions required to complete each subtask and total assigned times to develop a leveled time for the subtask. Documentation should demonstrate that the accuracy of the original data base has not been compromised in application or standard development.

◊ Total subtask times to develop a leveled time for the entire task.

- **Standard Time Data.**

  Standard time data (or elemental standard data) are developed for groups of motions that are commonly performed together, such as drilling a hole or painting a square foot of surface area. Standard time data can be developed using time studies or predetermined leveled times. After development, the analyst can use the standard time data instead of developing an estimate for the group of motions each time they occur.

  Typically, the use of standard time data improves accuracy because the standard deviations for groups of motions tend to be smaller than those for individual basic motions. In addition, their use speeds standard development by reducing the number of calculations required.

  Estimate development using standard time data is much like using predetermined leveled times except that groups of motions are estimated as a single element instead of individual body motions.

8.2 Identifying Elements of a Labor Standard (cont)

Leveled Time (cont)

- **Work Sampling.**

  Work sampling is commonly used to develop non-engineered standards. It cannot be used alone to develop engineered standards. However, it can be used to supplement or check standard development by more the definitive techniques described above. For example, it can be used to determine job content and assess productive vs. nonproductive time.

  In work sampling, analysis is based on a large number of random, rather than continuous observations. Estimates are based the proportion of time spent by one or more persons on a given activity. This is useful for jobs with irregular components that vary in the amount of time per unit of output.

  To use work sampling in standard development, the analyst must:

  - Identify and define activities involved in the work (through discussions with the workers and preliminary observations).
  - Develop the method(s) for observing and recording activities.
  - Determine the sampling strategy (e.g., stratified or unstratified) and number of observations (by time and place).
  - Record observed activities during each period.
  - Consolidate and analyze the data.
  - Use the data collected to develop nonengineered standards or to supplement development of engineered standards.

PF&D Allowance

After the leveled time is developed, estimators must consider a personal, fatigue, and delay (PF&D) allowance.

DOD-HDBK-345
DOD-5010.15-M

Contractor work measurement policies and procedures should provide the detailed rationale used for applying PF&D allowances. Each allowance should be identified and quantified. On a cumulative total basis, PF&D allowances typically total 15 percent. However, allowances may be higher or lower depending on the nature of the work and related working conditions. For example, strenuous work in an extremely hot environment would typically merit a higher PF&D allowance than light labor performed in an air conditioned room.

8.2 Identifying Elements of a Labor Standard (cont)

PF&D Allowance (cont)

DOD-5010.15.1-M, Basic Volume, Appendix II

- **Personal Allowance.**

  A personal allowance considers time for a worker to take care of personal needs, such as trips to the rest room and drinking fountain. Specific allowances should consider the surroundings, working conditions, and job requirements which cause the employee to stop work from time to time to attend to necessary personal needs. The table below delineates some of the elements that should be considered in personal allowance development.

PERSONAL ALLOWANCE CONSIDERATIONS

Personal allowance documentation should document:	**Considerations**	**Typical Percentage Allowance**
A Basic Allowance which considers the breaks available for work during an 8-hour day.	Most firms allow, at least, two 10-minute breaks during each 8-hour shift, the basic personal allowance is 4.2 percent (20 minutes/480 minutes).	4.2
Any allowance for working conditions.	Normal office conditions	0.0
	Normal shop, central heat, slightly dirty or greasy.	1.0
	Slightly disagreeable conditions. Exposed to inclement weather part of the time, poor heating, or poor cooling.	3.0
	Extremely disagreeable conditions. Proximity to hot objects, continuous exposure to disagreeable odors and fumes, or to excessive temperature ranges.	6.0
Any allowance at the beginning and/or end of each shift, to get out/put away tools and equipment, clean up work area, or to don/remove special clothing (e.g. aprons and smocks).	Total time allowed: 5 minutes 10 minutes 15 minutes 20 minutes	 1.0 2.1 3.1 4.2
Any allowance for work performed in a super-clean room.	An additional allowance may be added to consider the time require to assure that super- clean room requirements are met.	4.0

Be careful when contractors use predetermined time systems. Some predetermined time systems include a partial or complete allowance for PF&D. If the contractor uses such standards, additional PF&D consideration may not be appropriate.

8.2 Identifying Elements of a Labor Standard (cont)

PF&D Allowance (cont)

DOD-5010.15.1-M, Basic Volume, Appendix II

- **Fatigue Allowance.**

  A fatigue allowance considers the time required to recuperate from fatigue related to factors such as general working conditions, the nature of the work, and the health of the worker. The table below delineates some of the factors that analysts should consider in developing a fatigue allowance.

FATIGUE ALLOWANCE CONSIDERATIONS

Personal allowance documentation should document:	Considerations	Typical Percentage Allowance
Any allowance for handling heavy weights.	Effective Net Pounds Percent of Time Under Load <table><tr><td>Handled</td><td>1-12</td><td>13-25</td><td>26-50</td><td>51-75</td><td>76-100</td></tr><tr><td>1-10</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>11-20</td><td>1</td><td>3</td><td>5</td><td>7</td><td>10</td></tr><tr><td>21-30</td><td>2</td><td>4</td><td>9</td><td>13</td><td>17</td></tr><tr><td>31-40</td><td>3</td><td>6</td><td>13</td><td>19</td><td>25</td></tr><tr><td>41-50</td><td>5</td><td>9</td><td>17</td><td>25</td><td>34</td></tr><tr><td>51-60</td><td>6</td><td>11</td><td>22</td><td>x</td><td>x</td></tr><tr><td>61-70</td><td>7</td><td>14</td><td>28</td><td>x</td><td>x</td></tr><tr><td>71-80</td><td>8</td><td>17</td><td>34</td><td>x</td><td>x</td></tr></table> x – Study for possibilities for worker rotation and other means to relieve fatigue.	Select percentage from table.
	Multiply the table values above by the following factors to consider lifting requirements: For picking up from the floor, multiply the table value by 1.10. For placing the load above chest height, multiply table value by 1.20. For getting the load from chest height, multiply the basic allowance by 0.50.	Depends on work.
	For sliding and rolling objects, multiply the weight by the coefficient of friction to determine the effective weight moved. Coefficients of Friction (Average Values) <table><tr><td>Surfaces</td><td>Friction Coefficient</td></tr><tr><td>Wood on Wood</td><td>0.4</td></tr><tr><td>Wood on Metal</td><td>0.4</td></tr><tr><td>Metal on Metal</td><td>0.3</td></tr></table>	Depends on work.

8.2 Identifying Elements of a Labor Standard (cont)

PF&D Allowance (cont)

FATIGUE ALLOWANCE CONSIDERATIONS (CONTINUED)		
Personal allowance documentation should document:	**Considerations**	**Typical Percentage Allowance**
Any allowance for the position that workers must assume to perform the work.	Sitting or standing. (Work will normally be less tiresome if the position is varied frequently.)	0.0
	Sitting.	1.0
	Walking.	1.0
	Standing.	2.0
	Climbing or descending ramps, stairs, or ladder.	4.0
	Working in close cramped quarters.	7.0
Any allowance for the mental requirements of the job.	Work largely committed to habit (e.g., simple calculations on paper, reading easily understood material, counting and recording, simple inspection requiring attention but little discretion, or arranging papers by letter or number).	0.0
	Work requires full attention (e.g., copying numbers or instructions, remembering part number while checking a parts list, or filing papers by subject of familiar nature.	2.0
	Work requires concentrated attention (e.g., reading of nonroutine instructions or cross-checking items).	4.0
	Work requires deep concentration (e.g., making swift mental calculations or memorizing items).	8.0
Any allowance for the lighting on the job site.	Continual glare on work area. Work requiring constant change of light. Less than 75 foot candle power on work surface for normal work. Less than 125 foot candle power on work surface for close work.	2.0
Any allowance for noise on the job site.	Constant, rather loud noises over 60 decibels (e.g., machine shops or motor test shops).	1.0
	Average constant noises, level but with loud, sharp, intermittent, or staccato noise (e.g., nearby riveters or punch presses).	2.0
Any allowance to consider the fatigue resulting from fast, highly repetitive operation cycles. (Cycle Time is the time from one cycle start to the next.)	0.00 to 0.20 minute cycles	4.0
	0.21 to 0.40 minute cycles	3.0
	0.41 to 0.80 minute cycles	2.0
	0.81 to 2.50 minute cycles	1.0
	2.51 minutes or more	0.0

8.2 Identifying Elements of a Labor Standard (cont)

PF&D Allowance (cont)

FATIGUE ALLOWANCE CONSIDERATIONS (CONTINUED)		
Personal allowance documentation should document:	**Considerations**	**Typical Percentage Allowance**
Any allowance for the use of safety devices or clothing.	No allowance should be made here unless it is necessary to remove the equipment occasionally for relief or if wearing the item causes fatigue.	
	Face shield	2.0
	Rubber boots	2.0
	Goggles or welding mask	3.0
	Tight, heavy protective clothing	4.0
	Filter mask	5.0
	Safety glasses	0.0

DOD-5010.15.1-M, Basic Volume, Appendix II

- **Delay Allowance.**

  A delay allowance covers unavoidable, predictable, and unpredictable delays for such activities as replenishing materials, rejecting nonstandard parts, making minor equipment repairs, and receiving instructions. The delay allowance should not include time for rework or repair of substandard parts.

  In considering a specific delay allowance, the analyst must consider delays inherent in the job (e.g., supervisory interruptions, moving from one work station to another, or waiting for equipment), as well as, the relationship between each job and adjacent jobs. No delays that can be prevented by the employee should be considered.

8.2 Identifying Elements of a Labor Standard (cont)

PF&D Allowance (cont)

The table below delineates some of the elements that should be considered in delay allowance development.

DELAY ALLOWANCE CONSIDERATIONS		
Personal allowance documentation should document:	**Considerations**	**Typical Percentage Allowance**
Basic Allowance	Isolated job. Little coordination with adjacent jobs.	1.0
	Fairly close coordination with adjacent jobs.	2.0
Any allowance for balancing delay, where employees are required to move from one work station to another to balance work flow.	Worker moves once each 5 minutes.	5.0
	Worker moves once each 30 minutes.	3.0
	Worker moves once each 60 minutes.	2.0
	Worker moves once each 2 hours.	0.0

8.2 Identifying Elements of a Labor Standard (cont)

Special Allowances

Any proposed special allowance must be supported by detailed engineering analysis. An appropriate study should be conducted in each shop or functional area to ascertain any requirement for a separate delay allowance. The analyst should assure that there is no duplication between cycle time elements and allowance elements and that the Special Allowance does not become a dumping ground for operation activity that is not an integral part of shop work load.

- Work elements such as cleaning chips from equipment, tool care, or tool replacement, though occurring irregularly, should be measured and the time required prorated directly to the machine operating portion of the work cycle rather than as an allowance.
- Certain other irregularly occurring elements having a direct relationship to the job such as obtaining parts and materials and periodic inspection should be added to the cycle time on a prorated basis or as a separate work element rather than added as an allowance.

When a special allowance is appropriate, the time required is first calculated in minutes and then converted to a percentage. The base for calculating and applying the allowance percentage is normally the sum of the leveled time and the PF&D allowance. Appropriate special allowances typically fall into two categories:

- Those that consider elements that occur on an unforeseeable basis:
  - ◊ Power failures of nonreportable duration.
  - ◊ Minor repairs to defective parts.
  - ◊ Waiting for a job assignment.
  - ◊ Obtaining job information from a supervisor, inspector , or production control specialist.
  - ◊ Unsuccessful hunt for parts or materials.
  - ◊ Machine breakdown of nonreportable duration.
- Those that consider elements that occur periodically (daily, weekly, hourly) such as:
  - ◊ Cleaning and lubricating equipment.
  - ◊ Work area clean-up.

8.2 Identifying Elements of a Labor Standard (cont)

Applying an Allowance to Leveled Time

Allowances are normally expressed as a percentage of standard time spent unproductively (e.g., a 15 percent PF&D Allowance indicates that 15 percent of the worker's standard time is spent unproductively). To apply an allowance, the analyst must determine how much the leveled time must be increased to allow for the unproductive time. This is accomplished by dividing the leveled time by the percentage of time spent productively.

For example. The leveled time for a particular task is 170 minutes, the PF&D Allowance is 15 percent, and there is no special allowance. The standard time would be calculated as:

$$\text{Standard Time} = \frac{\text{Leveled Time}}{100\% - \text{Allowance Percentage}}$$

$$= \frac{170}{1.00 - .15}$$

$$= \frac{170}{.85}$$

$$= 200 \text{ Minutes}$$

Note that 85 percent of the standard time (85% of 200 = 170) is the leveled time. The remaining 15 percent of the standard time (15% of 200 minutes = 30 minutes) is the allowance for personal, fatigue, and delay factors.

8.3 Measuring and Projecting Operations Efficiency

Comparing Labor Standard with the Actual Time

Standards represent goals of efficient production. Production on the plant floor is rarely completed in the allowed standard time. Work Measurement Systems commonly use realization or efficiency factors to evaluate how the actual time required to complete a task compares with the standard time for that task. Analysts can then use these measures to identify tasks that require special analysis to identify and correct inefficient operations.

Since estimators strive to estimate realistic contract costs, they use realization or efficiency factors with labor standards to estimate future labor hours required to complete the task.

Calculating a Realization Factor

A realization factor is generally a measure of overall performance (e.g., shop, product line, or plant). It will normally be calculated from historical data as:

$$\text{Realization Factor} = \frac{\text{Total Actual Hours}}{\text{Earned Standard Hours}}$$

Don't be confused by the fact that some firms refer to this calculation as an efficiency factor.

In realization factor calculation, total actual hours include all operation touch labor hours (reconcilable to payroll hours) associated with the tasks represented by the standard hours in the denominator, including “lost time” or “idle time” accounts and “off standard” or “unmeasured” work.

For example. A task has a standard time of 1.5 hours. Actual time to perform the task 100 times is 300 hours. The realization factor would be calculated as follows:

$$\begin{aligned}\text{Realization Factor} &= \frac{\text{Total Actual Hours}}{\text{Earned Standard Hours}} \\ &= \frac{300 \text{ Actual Hours}}{1.5 \text{ Standard Hours} \times 100 \text{ Repetitions}} \\ &= \frac{300}{150} \\ &= 2.00\end{aligned}$$

In this case, actual experience shows that the task takes twice as many hours as the standard time indicates.

8.3 Measuring and Projecting Operations Efficiency (cont)

Developing an Estimate Using a Realization Factor

To develop a realistic estimate of actual hours, the estimator must consider both the standard time and the realization factor.

For example. An estimate of the actual time to complete the task above for 50 units would be calculated as:

$$\begin{aligned}\text{Estimate for the Task} &= \text{Standard Hours} \times \text{Repetitions} \times \text{Realization Factor} \\ &= 1.5 \times 50 \times 2.00 \\ &= 150 \text{ Labor Hours}\end{aligned}$$

Calculating an Efficiency Factor

An efficiency factor is calculated to demonstrate efficiency against the standard (e.g., a task with an efficiency factor of .60 is being performed at 60 percent efficiency). The factor is normally calculated:

$$\text{Efficency Factor} = \frac{\text{Earned Standard Hours}}{\text{Actual Hours}} \times 100$$

For example. A task has a standard of 1.8 hours. Actual time to perform the task 100 times is 400 hours. The efficiency factor would be calculated as follows:

$$\begin{aligned}\text{Efficiency Factor} &= \frac{\text{Earned Standard Hours}}{\text{Actual Hours}} \times 100 \\ &= \frac{1.8 \text{ Standard Hours} \times 100 \text{ Repetitions}}{400 \text{ Actual Hours}} \times 100 \\ &= \frac{180}{400} \times 100 \\ &= .450 \text{ or } 45.0 \text{ Percent}\end{aligned}$$

8.3 Measuring and Projecting Operations Efficiency (cont)

Developing an Estimate Using an Efficiency Factor

To develop a realistic estimate, the estimator must consider both the standard time and the efficiency factor.

For example. An estimate of the time to complete the task above for 50 units would be calculated as:

$$\text{Estimate for the Task} = \frac{\text{Standard Hours} \times \text{Repetitions for the Task}}{\text{Efficiency Factor}}$$

$$= \frac{1.8 \times 50}{.45}$$

$$= 200 \text{ Labor Hours}$$

Analyzing Realization and Efficiency Factors

Analysis of labor estimates developed using labor standards requires extensive knowledge and experience. Even skilled industrial engineers typically require special training in work measurement analysis. As a result, you should normally request technical support whenever an offeror estimates labor hours using labor standards.

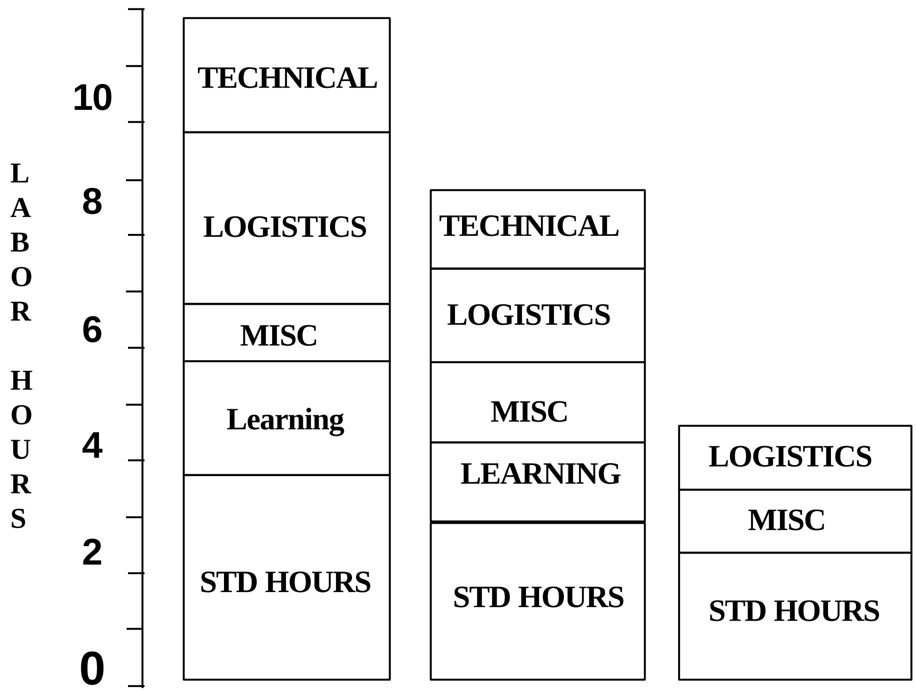
For each standard, offerors should be required to provide information on internal analyses of the variance between the actual time required to complete the work and the standard time to determine the causes for the variance and identify ways of managing performance improvement.

You should expect offeror's to demonstrate continued improvement in realization and efficiency factors. The figure below depicts some of the reasons for that improvement.

- At Unit #1, total labor-hours include substantial inefficiencies related to technical, logistics, learning, and other factors.
- As production increases, there should be reductions in all areas of inefficiency. In most cases, there should also be an improvement in the labor standard itself, as better production methods are identified and implemented.
- By Unit #1000, the contractor should be operating efficiently, with only minor inefficiencies related to such factors as unavoidable parts shortages.

8.3 Measuring and Projecting Operations Efficiency (cont)

Analyzing Realization and Efficiency Factors (cont)



Typical Variances from Standard during a Production Program

However, improvement will not automatically follow this pattern. Effective analysis and management effort are required. Even when these are present, improvement may be hampered by factors such as repeated changes in design or production methods.

Still, the goal of both the contractor and the Government should be continuous improvement. Even when operations are being performed at or close to standard, the contractor should be searching for methods improvements that will reduce costs and improve overall efficiency.

8.3 Measuring and Projecting Operations Efficiency (cont)

Projecting Realization and Efficiency Factors

Improvement curves and moving averages are commonly used to project variation from labor standards. Either technique can be acceptable depending on the situation. Technical assistance can be very valuable in evaluating offeror forecasts.

- **Improvement Curves.**

  Using an improvement curve to track and project variance from a labor standard assumes that the variance is related to the number of units produced. As more units are produced, the variance is expected to decline following improvement curve theory.

  Be cautious of any estimate for continuous production that does not consider variance reduction following the improvement curve. Continuous improvement is one of the reasons for using labor standards, because they provide detailed information on the areas that offer the greatest opportunity for improvement.

- **Moving Averages.**

  Firm's often use moving averages to track and project variance from a labor standard when the variance is not expected to follow an improvement curve. This assumption is often valid when product production is not continuous or there are frequent changes in design or production methods.

  While moving averages are an acceptable way to track and project variances, do not forget that a firm could be using a moving average to hide a downward trend in the data. That is particularly true in cases where the firm proposes a single moving average calculated over a large number of periods. Government technical personnel can provide invaluable assistance in assuring that averages are not masking a trend in the data and that data from one or two periods are not unduly affecting the average.

8.4 Identifying Issues and Concerns

Questions to Consider in Analysis

As you perform cost analysis, consider the issues and concerns identified in this section, whenever you use work measurement.

- **Is the offeror using available standards and realization or efficiency factors to estimate contract cost?**

  If the offeror has an active Work Measurement System, the System provides vital insight into direct labor costs. Information on related time standards and variance analysis are cost or pricing data and must be provided by the offeror whenever you require cost or pricing data. You should also consider requesting information on time standards and variance analysis, whenever you require an offeror to submit information other than cost or pricing data.

- **Were standards developed using appropriate process analysis and accepted methods of standard development?**

  Many firms refer to historical costs as standards. Using historical costs does not provide the methods analysis and engineering discipline normally associated with the use of engineered labor standards in estimating.

- **Are realization or efficiency factors based on experience with the same or similar products and processes?**

  In a cost proposal, either factor should consider experience with the same product or similar products and processes. A realization standard at one facility should not be used at another facility unless the conditions are the same. Do not compare realization factors for one facility with those for other facilities.

- **Are standards and factors current?**

  The data used to develop standards should be current and representative of current methods, facilities, and working conditions. Realization/efficiency factors should be based on the current standards. If you have questions or concerns, seek assistance from Government technical and audit representatives.

8.4 Identifying Issues and Concerns (cont)

Questions to Consider in Analysis (cont)

- **What efforts are being taken to control variance from labor standards?**

  Reasons for the differences between the standard hours and actual hours should be explained. Improvement curves are often used to estimate the reduction of variances from standard as production continues. Setting and achieving aggressive goals for improvement of realization or efficiency factors beyond historical improvement curve effects should be a prime factor in reviewing contractor performance.

- **How are rework and repair considered in the estimate?**

  Rework and repair occurs when a part or assembly is rejected in an inspection or test and sent back for correction of the deficiency. In addition, some completed parts and assemblies must be reworked to incorporate design changes. The cost of rework should not be included in the labor standard, related allowances, or the realization factor. Instead, time spent on rework should be accounted for separately. However, labor standards can be used in estimating the labor effort required for rework. Historical rework costs should be carefully screened to eliminate rework costs associated with one-time problems or changes.

- **Is the method used for realization or efficiency factor forecasting appropriate?**

  Use of improvement curves is appropriate in continuous production situations that should foster variance reduction. Use of moving averages is appropriate in situations where sporadic production or other factors hamper efforts to reduce any variance from standard.

CHAPTER 9

Net Present Value Analysis

Learning Objective

At the End of This Chapter	At the end of this chapter you will be able to: **Classroom Learning Objective 9/1** Correctly use net present value analysis in estimating and analyzing contract cost or price.

9.0 Chapter Introduction

In This Chapter

In this chapter, you will learn to use net present value analysis in cost and price analysis.

9.0 Chapter Introduction (cont)

Time Value of Money	The time value of money is probably the single most important concept in financial analysis. When we say that money has time value, we mean that a dollar to be paid (received) today is worth more than a dollar to be paid (received) at any future time. Money has a time value because of the opportunity to earn interest or the cost of paying interest on borrowed capital. For example, assume that you need to buy a new car but do not have the money that you need to pay for it. You must borrow the entire purchase price. Two dealers offer to sell you identical cars for $21,000. Dealer #1 requires cash on delivery. Dealer #2 will provide you an interest-free loan for one year. Where would you buy the car? Probably from Dealer #2, because you will save all the interest for the first year of ownership.
Present Value	In the example above, Dealer #2 was clearly the low-cost choice (because of the interest-free loan for one year), but what if Dealer #1 offered the car at a lower price, say $20,000? Which would be the low-cost choice then? To make that decision, you must be able to determine the present value of each alternative. If you could invest $20,000 at 5.0 percent interest, it would be worth $21,000 at the end of one year. Based on that calculation, we could say that $20,000 is the present value of $21,000 one year from now when the interest rate is 5.0 percent. At that interest rate, you would presumably be indifferent about where to buy your car because the present value of the two choices is the same.

9.0 Chapter Introduction (cont)

Net Present Value

Calculating present value may involve receipts as well as expenditures. For example, the alternatives may have some salvage value after their useful life has ended. The estimated receipt from the sale of the item must be considered in your analysis. The difference between the present value of the receipts and the present value of the expenditures is net present value. The best financial choice is the alternative with the highest net present value. In procurement, the alternative with the highest net present value is the alternative with the smallest payment net present value.

Factors Affecting Net Present Value

The major factors affecting present value are the timing of the expenditure (receipt) and the discount (interest) rate. The higher the discount rate, the lower the present value of an expenditure at a specified time in the future. For example, as you learned above, $20,000 is the present value of $21,000 one year from now when the interest rate is 5.0 percent. If the interest rate were 10.0 percent, $19,090.90 would be the approximate present value of $21,000 one year from now.

Note that the change in the interest rate would have a significant affect on your net present value analysis in the car case. Your choice is still to pay $20,000 now or $21,000 a year from now. At an interest rate of 10 percent you could invest $19,090.90 today to earn the $21,000 a year from now. So it appears that the low-cost choice is to wait and pay the $21,000 in one year.

Office of Management and Budget (OMB) Circular No. A-94, Guidelines and Discount Rates for Benefit-Cost Analysis of Federal Programs, delineates the rates that you should use in Government net present value analysis.

Net Present Value Analysis

Regardless of the application, you should use this 5-step process in net present value analysis:

Step 1. Select the discount rate.

Step 2. Identify the costs/benefits to be considered in analysis.

Step 3. Establish the timing of the costs/benefits.

Step 4. Calculate net present value of each alternative.

Step 5. Select the offer with the best net present value.

9.0 Chapter Introduction (cont)

Lease-Purchase Analysis Examples

In this chapter, we will demonstrate the application of net present value analysis concepts using lease-purchase examples. Our use of these examples is not meant to ignore other uses of net present value analysis in Government contracting. We selected the lease-purchase decision because of the emphasis in OMB Circular No. A-94 and because of the growing Government interest in leasing as a viable alternative to purchase.

9.1 Identify Situations for Use

OMB Suggested Use

OMB Circular No. A-94, Para 4

FAR 23.203

Unless precluded by agency procedures, OMB suggests the use of net present value analysis in any analysis to support Government decisions to initiate, renew, or expand programs or projects which would result in a series of measurable benefits or costs extending for three or more years into the future. Examples of acquisition decisions that involve such analyses include:

- Lease-purchase analyses
- Analyses of different lease alternatives
- Life-cycle cost analyses
- Analyses of acquisition costs and energy-utilization costs whenever the results would be meaningful, practical, and consistent with agency programs and needs.

Required Lease-Purchase Analysis

OMB Circular No. A-94, Para 13

In addition to the suggested application to any benefit-cost analysis, OMB Circular No. A-94 **requires** that any decision to lease a capital asset be justified as preferable to direct Government purchase and ownership in situations where **both** the following are true:

- The lease-purchase analysis concerns a capital asset or a group of related assets whose total fair market value exceeds $1 million.
- The lease-purchase analysis concerns a capital asset (including durable goods, equipment, buildings, facilities, installations, or land) which is:
  - ◊ Leased to the Government for a term of three or more years;
  - ◊ New, with an economic life of less than three years, and leased to the Government for a term of 75 percent or more of the economic life of the asset;
  - ◊ Built for the express purpose of being leased to the Government; or
  - ◊ Leased to the Government and clearly has no alternative commercial use (e.g., a special-purpose Government installation).

9.1 Identify Situations for Use (cont)

Required Lease-Purchase Analysis (cont)

The analysis conducted in support of that justification should involve net present value analysis and can be performed in one of three ways, as delineated in the table below:

Methods of Lease-Purchase Analysis	USE...
Conduct a separate lease-purchase analysis for each acquisition.	Only for major acquisitions. A lease is a major acquisition when one of the following is true: • Acquisition is a separate line item in the Agency's budget. • The agency or the OMB determines that the acquisition is a major one. • The total purchase price of the asset or group of assets will exceed $500,000.
Conduct periodic lease-purchase analysis of the recurring acquisition of assets for the same general purpose.	For an entire class of assets.
Adopting a policy for smaller leases and submitting the policy to OMB for approval.	Normally after the agency demonstrates that: • The leases in question would generally result in substantial savings to the Government. • The leases in question are so small or so short as to make separate analyses impractical. • Leases of different types are scored consistent with the requirements of OMB Circular No. A-11, Preparation and Submission of Annual Budget Estimates.

9.2 Selecting A Discount Rate

OMB Discount Rate Guidance

Unless precluded by agency practice, you should use the discount rates contained in Appendix C of OMB Circular A-94. These rates are based on the rate that the Treasury Department pays to borrow money for periods from 91 days to 30 years and they are updated annually at the time of the President's budget submission to Congress. Rate updates are available upon request from the OMB Office of Economic Policy.

Nominal Treasury Rates

OMB Circular No. A-94, Appendices A & C

For most benefit-cost analysis you should use nominal discount rates (i.e., discount rates that include the effect of actual or expected inflation/deflation). The following rates are the actual rates contained in OMB Circular A-94, Appendix C for use through February 1997:

NOMINAL DISCOUNT RATES	
Maturity in Years	**Discount Rate**
[2] 3 years	5.4%
5-year	5.5%
7-year	5.5%
10-year	5.6%
[3] 30 years	5.7%

In benefit-cost analysis, use the interest rate for the period of the analysis (e.g., use 5.4 percent to discount all expenditures/receipts for a 3-year lease analysis). To analyze a project with expenditures with periods different from those presented above, use linear interpolation to determine the appropriate discount rate. Linear interpolation assumes that the change is the same for each year between known points. The example below demonstrates the steps involved in interpolating an interest rate for evaluating an 8-year lease:

9.2 Selecting A Discount Rate (cont)

Nominal Treasury Rates (cont)

Step 1. Estimate the change for each year between a 7-year maturity and a 10-year maturity:

$$\begin{matrix}\text{Amount Rate} \\ \text{Changes Each Year}\end{matrix} = \frac{\begin{pmatrix}\text{Next Higher} \\ \text{Maturity Period Rate}\end{pmatrix} - \begin{pmatrix}\text{Next Lower} \\ \text{Maturity Period Rate}\end{pmatrix}}{\begin{pmatrix}\text{Next Higher} \\ \text{Maturity Period}\end{pmatrix} - \begin{pmatrix}\text{Next Lower} \\ \text{Maturity Period}\end{pmatrix}}$$

$$= \frac{\text{10 - Year Rate} - \text{7 - Year Rate}}{\text{10 Years} - \text{7 Years}}$$

$$= \frac{5.6 - 5.5}{10 - 7}$$

$$= \frac{.1}{3}$$

$$= .033$$

Step 2. Add the estimated percent change for each year over the earlier maturity to calculate the interpolated rate.

An 8-Year maturity is one year longer than a 7-Year maturity.

$$\text{Interpolated Rate} = \begin{pmatrix}\text{Next Lower} \\ \text{Maturity Period Rate}\end{pmatrix} + \begin{pmatrix}\text{Rate Change} \\ \text{for Each Year}\end{pmatrix} \text{ x (Added Years)}$$

$$= 5.5 + (.033)(1)$$

$$= 5.5 + (.033)$$

$$= 5.533$$

9.2 Selecting A Discount Rate (cont)

Real Treasury Rate

OMB Circular No. A-94, Appendices A & C

In some projects (e.g., long-term real estate leases), you may find it more reasonable to state payments in terms of stable purchasing power (that is, constant dollars) and adjust them separately using a pre-determined price index. In such situations, cash flows should be discounted using the real Treasury borrowing rate for debt of comparable maturity. The real Treasury rate is the nominal Treasury rate adjusted to eliminate the effect of anticipated inflation/deflation. These rates are also contained in Appendix C of OMB Circular A-94 and are updated annually. The following real rates are to be used for discounting dollar cash flows through February 1997.

REAL DISCOUNT RATES	
Maturity in Years	**Discount Rate**
≤ 3 years	2.1%
5 year	2.3%
7 year	2.5%
10 year	2.7%
≥ 30 years	2.8%

You should use the linear interpolation process described above to determine the appropriate discount rate to use in analyzing a project with an analysis period different from those presented above.

9.3 Identifying Cash Flows to Consider

Section Introduction

A cash flow is a receipt or expenditure related to the proposed lease or purchase. Guidance on the costs/benefits that you should consider in lease-purchase analysis is provided in both FAR and OMB Circular No. A-94. The solicitation should require each offeror to identify all the relevant cash flows involved with its proposal. Remember, the purpose of the evaluation is to identify the best net present value for the Government.

Analysis Period

OMB Circular No. A-94, 13c(8)

In lease-purchase analysis, the proper period for analysis is the lease period including all renewal options. The period of the projected lease must be defined in the solicitation to assure identification and analysis of all relevant cash flows.

Points to Consider in Identifying Costs and Benefits for Analysis

OMB Circular No. A-94, 13c(1)

OMB Circular No. A-94, 6a(1)

Lease-purchase analysis should compare the net present value of the incremental costs related to leasing the asset with the incremental costs related to purchasing (or constructing) and owning the asset. You should consider incremental costs associated with acquisition as well as the ancillary costs related to acquisition and ownership. Use the following general guidelines as you identify incremental benefits and costs to include in your analysis:

- Analysis should consider costs or benefits associated with one alternative in the evaluation of other alternatives. For example, if the lease payments include maintenance, the purchase alternative should also include the cost of maintenance.
- Analysis should consider costs or benefits that will be different for different alternatives. For example, if different alternatives will use substantially different amounts of electricity, the cost of electricity should be considered.
- Analysis should not consider sunk costs or benefits. Past experience is relevant only in helping to estimate future costs or benefits. For example, if the Government has decided to replace existing equipment, the value of that equipment is not relevant.
- Analysis should not consider costs which will be identical for all alternatives. For example, if the Government has decided to replace existing equipment, the cost of removing that equipment is not relevant because it must be accomplished for all alternatives.

9.3 Identifying Cash Flows to Consider (cont)

Examples of Lease-Purchase Costs and Benefits Commonly Considered

Lease-purchase analysis is one area where you might be required to use net present value analysis. The costs and benefits identified below for lease-purchase analysis demonstrate the type of cash flows that you should consider in a net present value analysis.

OMB Circular No. A-94, 13c(3) FAR 7.401

- **Net Purchase Price.**

  Any net present value analysis of a decision to purchase an asset must consider the purchase price. OMB defines the purchase price of the asset as the price a willing buyer could reasonably expect to pay a willing seller in a competitive market to acquire the asset. Normally, lease-purchase decisions do not consider trade-ins of existing equipment. Disposal of existing equipment should be handled following agency property disposal procedures and considered as part of disposal costs and salvage value as presented below.

- **Lease Payments.**

  Any decision to lease property using Net present value analysis must consider the amount and timing of lease payments.

- **Ancillary Services.**

FAR 7.401

  If ancillary costs differ between alternatives, they should be considered. (If costs and timing are the same for all alternatives, they need not be considered.) Both OMB Circular No. A-94 and the FAR provide guidance on the ancillary costs and benefits that you should consider in lease-purchase analysis. The following points combine the recommendations from both sources:

OMB Circular No. A-94, 13c(5) and FAR 7.401

◊ All costs associated with acquiring the property and preparing it for use including:
  - Costs.
  - Transportation.
  - Installation.
  - Site preparation.
  - Design.
  - Management.

◊ Repair and improvement costs.
  - Estimated unplanned service calls.
  - Improvements projected to be required during the lease period to assure continued operation.

9.3 Identifying Cash Flows to Consider (cont)

Examples of Lease-Purchase Costs and Benefits Commonly Considered (cont)

- ◊ Operation and maintenance costs.
  - – Operating labor and supply requirements.
  - – Routing maintenance.
- ◊ Disposal Costs and Salvage Value.
  - – Modification required to return related equipment to its original configuration.
  - – Modification requirement to return related facilities to their original configuration.
  - – Equipment value to the Government at the end of the lease period (e.g., salvage value).

9.4 Determining Cash Flow Timing

In This Section

The timing of cash flows is a vital element of any net present value analysis. This section presents two methods for considering that timing.

General Equation for Present Value Calculation

You can compute the present value of any cash flow (expenditure/receipt) in the future, by multiplying the amount by the appropriate discount rate:

$$PV = DF(CF)$$

Where:

PV = Present value
DF = Discount factor
CF = Cash flow

Discount Factors

The discount factor that you use in net present value analysis will depend on the discount rate that you use and the timing of the cash flow. In defining the timing of the cash flow, you must identify the year and the timing during the year. There are three commonly used assumptions about when during the year the payment occurs:

- End-of-year payment—use this assumption when a single payment is made at the end of the year or the beginning of the year.
  - ◊ A payment that is due immediately is not discounted.
  - ◊ A payment that is due at the beginning of Year t is evaluated as a payment due at the end of Year t-1. For example, payments due at the beginning of Year 2 and Year 3 will be treated as if they are due at the end of Year 1 and Year 2.
- Mid-year payment—use when a single payment will be made mid-year or payments will be made monthly throughout the year.

9.4 Determining Cash Flow Timing (cont)

Offer-Identified Cash Flows

Solicitations must require all offerors to clearly define the amount and timing of each cash flow (expenditure/receipt) unique to the proposal. The proposal should also include a rationale to support the timing of any cash flow unless the timing is set forth in the contract.

For example. The timing of lease payments does not require any additional support because the timing (e.g., monthly, quarterly, or annually) is defined in the lease agreement. However, the lease agreement may include additional charges (e.g., on-call equipment repair). For such charges, the rationale for both the expenditure and its timing should be clearly defined in the proposal.

Government-Identified Cash Flows

Government technical personnel must identify cash flows related to different proposals that are beyond the control of the offerors.

For example. The amount and timing of expenditures related to Government ownership must also be identified prior to proposal evaluation. Normally, Government personnel will be responsible for preparing these estimates based on available information. However, each offeror may be required to provide information required to develop these estimates (i.e. recommended service schedules).

9.4.1 Discount Factors for End-of-Year Payment

When to Use End-of-Year Discount Factors

Use end-of-year discount factors when payments are due at the end of the year or the beginning of the year. Remember, that a payment due at the beginning of Year 3 is the same as a payment due at the end of Year 2.

End-Of-Year Discount Factor Calculation

The discount factor formula for end-of-year cash flow (payment/receipt) is written:

$$DF = \frac{1}{(1 + i)^t}$$

Where:

DF	=	the discount factor
i	=	the discount rate
t	=	the number of years until the payment (receipt is due)

For Example: Determine the present value (PV) of a payment of $1,000 due at the end of 1 year using the nominal discount rate for three years or less, 5.4 percent.

Discount Factor Calculation:

$$\begin{aligned} DF &= \frac{1}{(1 + i)^t} \\ &= \frac{1}{(1.054)^1} \\ &= \frac{1}{(1.054)} \\ &= .9488 \end{aligned}$$

Present Value Calculation:

PV = DF(CF)

= .9488($1,000)

= $949 (rounded to the nearest dollar)

A present value of $949 means that we would be theoretically indifferent to a payment of $949 now and a payment of $1,000 at the end of one year.

9.4.1 Discount Factors for End-of-Year Payment (cont)

Sum Factors for Repetitive End-of-Year Cash Flows

When there is a repetitive cash flow such as a lease payment, you can use sum factors to speed the calculation process.

For example. Determine the present value of a series of three payments of $1,000 each due at the end of each of the next three years, when the discount rate is 5.4 percent.

Year	Payment	Formula	Calculation	Discount Factor (DF)	Present Value (PV)
1	$1,000	$1/(1.054)^1$	1/1.054	.9488[a]	$ 949[b]
2	1,000	$1/(1.054)^2$	1/1.1109	.9002	$ 900
3	1,000	$1/(1.054)^3$	1/1.1709	.8540	$ 854
			Total	2.7030	$2,703

[a] Factors are rounded to the four decimal places.
[b] Amounts are rounded to the nearest dollar.

The present value of a series of three $1,000 end-of-year payments is $2,703, when the discount rate is 5.4 percent. This means that we would be theoretically indifferent between a single payment of $2,703 now and a series of three end-of-year payments of $1,000 beginning one year from now.

The present value of the series of three payments can also be computed by multiplying the amount of the payment ($1,000) by the Sum Factor (Total) from the Discount Factor column (2.7030):

$$PV = SF(CF)$$

Where:

PV = present value
SF = Sum Discount Factor
CF = Cash Flow

Calculations:

2.7030($1,000) = $2,703 present value (rounded to the nearest dollar)

9.4.1 Discount Factors for End-of-Year Payment (cont)

End-of-Year Nominal Discount Tables

Appendix A-1, Discount Factors--Nominal Rates, End-of-Year Payments, contains factors for the 3-year, 5-year, 7-year, 10-year, and 30-year discount rates.

End-of-Year Real Discount Tables

Appendix A-3, Discount Factors--Real Rates, End-of-Year Payments, contains factors for the 3-year, 5-year, 7-year, 10-year, and 30-year discount rates.

9.4.2 Discount Factors for Mid-Year Payment

When to Use Mid-Year Discount Factors

Use mid-year discount factors when a single payment will be made mid-year or payments will be made monthly throughout the year.

Mid-Year Discount Factor Calculation

The discount factor formula for mid-year cash flow (payment/receipt) is written:

$$\text{MYDF} = \frac{1}{(1 + i)^{[t - .5]}}$$

Where:

MYDF = the discount factor
i = the discount rate
t = the number of years until the payment (receipt) is due

For example. Determine the present value of a series of 12 monthly payments of $1,000 each due at the beginning of each month for 1 year. The total amount for the year is $12,000. These payments are spaced evenly over the year; hence the use of a MYDF would be appropriate.

Discount Factor Calculation:

$$\begin{aligned}\text{MYDF} &= \frac{1}{(1 + i)^{[t - .5]}} \\ &= \frac{1}{(1.054)^{[1 - .5]}} \\ &= \frac{1}{(1.054)^{.5}} \\ &= \frac{1}{(1.0267)} \\ &= .9740\end{aligned}$$

So the present value, PV is $12,000 x .9740 = $11,688

9.4.2 Discount Factors for Mid-Year Payment (cont)

Sum Factors for Repetitive Mid-Year Cash Flows

When there is a repetitive cash flow such as a lease payment, you can use sum factors to speed the calculation process.

For example. Determine the present value of a series of 36 monthly payments of $1,000 each due at the beginning of each month for the next three years; that is, $12,000 per year for three years. These payments are spaced evenly over the year; hence the use of a MYDF would be appropriate.

Year	Payment	Formula	Calculation	Discount Factor (MYDF)	Present Value (PV)
1	$12,000	$1/(1.054)^{.5}$	1/1.0267	.9740[a]	$11,688[b]
2	$12,000	$1/(1.054)^{1.5}$	1/1.0821	.9241	$11,089
3	$12,000	$1/(1.054)^{2.5}$	1/1.1405	.8768	$10,522
			Total	2.7749	$33,299

[a] Factors are rounded to the four decimal places.

[b] Amounts are rounded to the nearest dollar.

The present value of a series of three $12,000 mid-year payments is $33,299, when the discount rate is 5.4 percent. The present value of the series of three payments can also be computed by multiplying the amount of the payment ($12,000) by the Mid-Year Sum Factor (Total) from the Discount Factor column (2.7749):

$$PV = MYSF(CF)$$

Where:

PV = present value
MYSF = Mid-Year Sum Factor
CF = Cash Flow

Calculations:

2.7749($12,000) = $33,299 present value (rounded to the nearest dollar)

9.4.2 Discount Factors for Mid-Year Payment (cont)

Mid-Year Nominal Discount Tables	Appendix A-2, Discount Factors—Nominal Rates, Mid-Year Payments, contains factors for the 3-year, 5-year, 7-year, 10-year, and 30-year discount rates.
Mid-Year Real Discount Tables	Appendix A-4, Discount Factors—Real Rates, Mid-Year Payments, contains factors for the 3-year, 5-year, 7-year, 10-year, and 30-year discount rates.

9.5 Calculating Net Present Value and Selecting the Best Alternative

Net Present Value Analysis

Remember from the Chapter Introduction that you should use the following 5-step process in net present value analysis:

Step 1. Select the discount rate.

Step 2. Identify the costs/benefits to be considered in analysis.

Step 3. Establish the timing of the costs/benefits.

Step 4. Calculate net present value of each alternative.

Step 5. Select the offer with the best net present value.

In this section, we will demonstrate the use of that 5-step process in two lease-purchase decision examples using **nominal discount rates**. You would follow the same steps for any net present value analysis whether you are using **nominal discount rates or real discount rates**.

9.5 Calculating Net Present Value and Selecting the Best Alternative (cont)

Lease-Purchase Decision Example 1

Assume that you want to determine which of the following proposals will result in the lowest total cost of acquisition?

Offeror A—Proposes to lease the asset for 3 years. The annual lease payments are $10,000 per year. The first payment will be due at the beginning of the lease, the remaining two payments are due at the beginning of Years 2 and 3.

Offeror B—Proposes to sell the asset for $29,000. It has a 3-year useful life. At the end of the 3-year period it will have a $2,000 salvage value.

Step 1. Select the discount rate. The term of the lease analysis is three years, so we will use the nominal discount rate for three years, 5.4 percent.

Steps 2 and 3. Identify and establish the timing of the costs/benefits to be considered in analysis. The expenditures and receipts associated with the two offers and their timing are delineated in the table below: (Parentheses indicate a cash outflow.)

OFFER EXPENDITURES/RECEIPTS		
t	**Offer A**	**Offer B**
0	($10,000)	($29,000)
1	($10,000)	0
2	($10,000)	0
3	0	$2,000

9.5 Calculating Net Present Value and Selecting the Best Alternative (cont)

Lease-Purchase Decision Example 1

Step 4. Calculate net present value. The tables below summarize the net present value calculations applied to each alternative.

NET PRESENT VALUE OF OFFER A			
t	**Cash Flow**	**DF**	**PV**
0	($10,000)	1.0000	($10,000)
1	($10,000)	0.9488	($ 9,488)
2	($10,000)	0.9002	($ 9,002)
		Net Present Value	($28,490)

NOTE the following points in the net present value calculations above:

- There are no cash inflows associated with Offer A, only outflows.
- Payments due now are not discounted.
- Offeror A payments due at the beginning of Years 2 and 3 are treated as if they are due at the end of Years 1 and 2 (nominal rate, end of year from Appendix A-1).
- You could have calculated the net present value of Offer A using the Sum of Discount Factors (Appendix A-1) for the payments due at the beginning of Years 2 and 3. Remember that payments due now are not discounted and payments due at the beginning of Years 2 and 3 are treated as if they are due at the end of Years 1 and 2. The calculations would be: (Negative numbers indicate cash outflows)

NPV = -$10,000 + 1.8490(-$10,000)

= -$10,000 - $18,490

= -$28,490

9.5 Calculating Net Present Value and Selecting the Best Alternative (cont)

Lease-Purchase Decision Example 1 (cont)

NET PRESENT VALUE OF OFFER B			
t	**Cash Flow**	**DF**	**PV**
0	($29,000)	1.0000	($29,000)
3	$2,000	0.8540	$ 1,708
		Net Present Value	($27,292)

NOTE the following points in the net present value calculations above:

- Offer B salvage value is treated as a cash inflow at the end of Year 3 (Appendix A-1 discount factor).
- Payments due now are not discounted.

Step 5. Select the offer with the best net present value.
In this example, we would select Offer B, the offer with the **smallest negative net present value**.

9.5 Calculating Net Present Value and Selecting the Best Alternative (cont)

Lease-Purchase Decision Example 2

Assume that we want to determine which of the following proposals will result in the lowest acquisition cost?

Offeror A—Proposes to lease the asset for 3 years. The monthly lease payments are $1,500; that is, the total amount for each year is $18,000. These payments are spaced evenly over the year, so the use of a MYDF would be appropriate.

Offeror B—Proposes to sell the asset for $56,000. It has a 3-year useful life. At the end of the 3-year period it will have a $3,000 salvage value.

Step 1. Select the discount rate. The term of the analysis is three years, so we will use the nominal discount rate for three years, 5.4 percent.

Steps 2 and 3. Identify and establish the timing of the costs/benefits to be considered in analysis. The expenditures and receipts associated with the two offers and their timing are delineated in the table below:

OFFER EXPENDITURES/RECEIPTS		
t	**Offer A**	**Offer B**
0	0	($56,000)
1	($18,000)	0
2	($18,000)	0
3	($18,000)	$3,000

9.5 Calculating Net Present Value and Selecting the Best Alternative (cont)

Lease-Purchase Decision Example 2 (cont)

Step 4. Calculate net present value. The tables below summarize the net present value calculations applied to each alternative.

NET PRESENT VALUE OF OFFER A			
t	**Cash Flow**	**DF**	**PV**
1	($18,000)	0.9740	($17,532)
2	($18,000)	0.9241	($16,634)
3	($18,000)	0.8768	($15,782)
		Net Present Value	($49,948)

NOTE the following points in the net present value calculations above:

- There are no cash inflows associated with Offer A, only outflows.
- Offeror A payments are due monthly, so we used the nominal rate, mid-year factors from Appendix A-2 (see pg 9-22).
- You could also calculate the net present value of Offer A using the Sum of Discount Factors in Appendix A-2.

NPV = 2.7749(-$18,000)

= -$49,948

9.5 Calculating Net Present Value and Selecting the Best Alternative (cont)

Lease-Purchase Decision Example 2 (cont)

NET PRESENT VALUE OF OFFER B			
t	**Cash Flow**	**DF**	**PV**
0	($56,000)	1.0000	($56,000)
3	$3,000	0.8540	$ 2,562
		Net Present Value	($54,438)

NOTE the following points in the net present value calculations above:

- Offer B salvage value is treated as a cash inflow at the end of Year 3 (Appendix A-1 discount factor)(see pg 9-2)).
- Payments due now are not discounted.

Step 5. Select the offer with the best net present value.
In this example, we would select Offer A, the offer with the **smallest negative net present value**.

9.6 Identifying Issues and Concerns

Questions to Consider in Analysis

As you perform price/cost analysis, consider the issues and concerns identified in this section, whenever you use net present value analysis.

- **Is net present value analysis used when appropriate?**

  Net present value analysis should be used in any analysis supporting Government decisions to initiate, renew, or expand programs or projects which would result in a series of measurable benefits or costs extending for three or more years into the future.

- **Are the dollar estimates for expenditures and receipts reasonable?**

  The base for all present value calculations are estimates of future cash flows. The rationale for those estimates must be documented and supported just like any cost estimate. This includes estimates of costs that will be included in the contract or lease agreement and estimates of other cash flows that are not included. All will be used in present value calculations.

- **Are the times projected for expenditures and receipts reasonable?**

  Discount factors depend on the interest rate and the timing of the cash flow. The timing of any cash flow not documented in the contract or lease agreement must be clearly supported. The offeror is responsible for estimating and defending cash flow estimates included in the proposal. Government technical personnel have that responsibility for estimated costs related to item ownership.

- **Are the proper discount rates used in the net present value calculations?**

  Unless precluded by agency policy, discount rates should be taken from Appendix C of OMB Circular No. A-94. If they are not, the rationale must be documented. The rate selected should be for the number of time periods included in the analysis. If the period of the analysis does not match any of the discount rate periods delineated in OMB Circular No. A-94, linear interpolation should be used to estimate a rate for that period of time. Nominal discount rates should be used for any analysis not based on constant year dollars. Real discount rates should be used for any analysis that is based on constant year dollars.

9.6 Identifying Issues and Concerns (cont)

Questions to Consider in Analysis (cont)

- **Are the proper discount factors used in analysis?**

  End-of-year discount factors (Appendices A-1 and A-3) should be used for cash flows at the beginning or end of the year. Mid-year discount factors (Appendices A-2 and A-4) should be used for cash flows in the middle of the year or spread throughout the year (e.g., monthly or quarterly).

- **Are discount factors properly calculated from the discount rate?**

  End-of-year or mid-year discount rates should be calculated following the procedures delineated in Section 9.4.

- **Have all cash flows been considered?**

  Net present value analysis must consider all relevant cash flows throughout the decision life cycle.